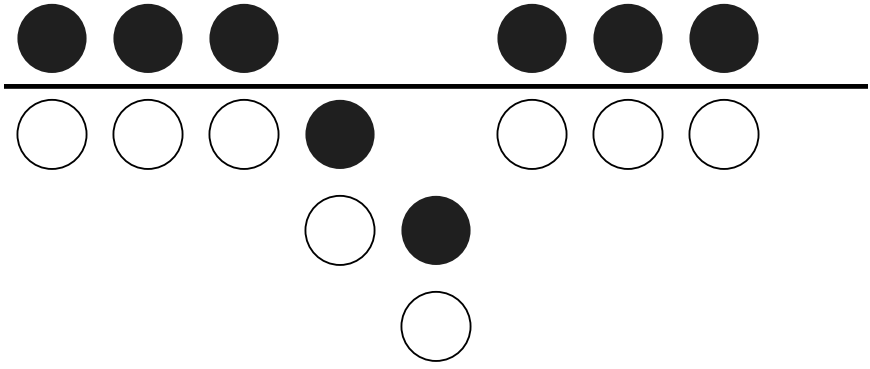
PROGRESSIVE MARGRITES



Volume 1

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Introduction

Progressive Margrites is a journal for the development of ideas and strategies in Margrites. Its purpose is to give new ways of understanding the game a place where they can be set out in full, tested against positions and games, and taken up by players who may extend or correct them. This first volume begins with the game itself and with the one idea on which its strategy so far rests.

Margrites from the First Move introduces the game to a player who has never seen it. It sets out the rules and the first skills, describes the consequences of the rules that most surprise new players, and ends with an annotated game.

Arithmetic Margrites sets out the first foundational idea in the game's development: that a game is decided by which side will reach the score it needs in fewer turns, and that formations, walls, support and captures matter only for what they do to that count. Strategy, as the game is understood today, is organized around this idea, and no successor to it has yet been articulated.

The next two articles work within the arithmetic view. *When to Capture* uses it to decide when a capture should be made, by the side ahead and by the side behind, in each phase of the game. *The Level Lock* applies it to the position in which two full lines meet at the center, and shows that when both sides come straight up to the line, the side that must open it is the side that started ahead.

The Waiting Game looks beyond that view. It follows a side that declines the race altogether and plays instead to hold a wall, take what comes against it and score at the end, and it records where that plan succeeds and where it fails. It is a first attempt to find a way of playing that the count does not already describe.

This journal exists to progress understanding of the game. It welcomes work that extends what is now understood, corrects it, or replaces it with something better.

The Editorial Board

Margrites from the First Move

An introduction for the new player

Margrites is played by two people on a board of nine files and eight rows, with eight pearls a side. Each player takes four steps a turn, pearls are captured by being outnumbered, and the player who gets more pearls across the board wins.

This article introduces the game to those who have not played it. It sets out the rules, the first skills, some consequences of the rules that surprise players when they begin, and a method of telling who is ahead. It closes with an annotated game and a set of exercises. Three subjects that are only introduced here, the count, the capture and the standoff at the center, have articles of their own in this volume.

The article is in eight parts.

1. *The game.* The board, the pearls, the turn of four steps, capture, and how a game ends.
2. *First skills.* Setting up, using a turn well, keeping pearls together, and making and avoiding captures.
3. *Surprises.* Eight things the rules allow that a new player does not expect.
4. *Counting.* How to tell who is ahead without guessing, and how to read the table of figures beneath a diagram.
5. *Ten mistakes.* The errors that lose most early games.
6. *A game.* A complete game of twelve turns a side, with notes.
7. *Exercises.* Eight questions, with answers.
8. *Where to go next.*

Part One
The Game

1. The Character of the Game

In most board games a player does one thing and then waits. In Margrites a player takes four steps in a turn, with any pearls, in any order, and only then does the opponent reply. A turn is a small plan carried out in full.

That one fact gives the game its character. A pearl in the middle of the board at the start of a turn can be off the far edge, and scored, by the end of it. A pearl that looked safe can be surrounded and removed before its owner can do anything. Two players can look at the same quiet position and one of them can be four steps from winning.

The rules are short. There is one kind of piece and one kind of step, and capture is a single rule about numbers. Everything in this Part can be learned in ten minutes.

What the rules produce is a race and a fight at the same time, and the two pull against each other. Points are scored only by getting pearls across the board. Captures score nothing. Every capture does change the race, sometimes in favor of the side that lost the pearl. A player has to decide, turn after turn, whether to run or to fight, and the right answer is often not the obvious one.

The rules also have consequences that are hard to foresee. A player can make six captures and lose. A player can give a pearl away on purpose and win by doing it. Five pearls can close the whole board, and two can open it again. None of this needs special rules. It all follows from the few rules in this Part, and Part Three sets out eight such cases.

2. The Board and the Pearls

The board has nine files and eight rows. The files are lettered a to i from left to right. The rows are numbered 1 to 8 from the top. A square is named by its file and its row, so e4 is the square on the e-file and the fourth row.

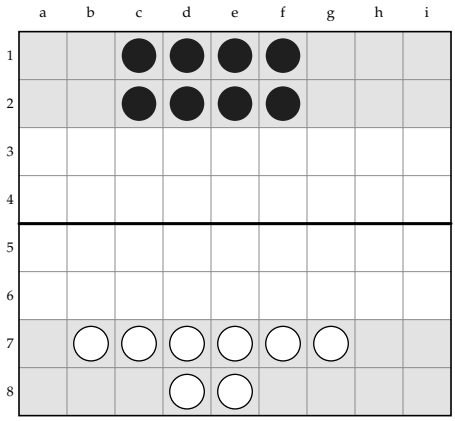

DIAGRAM 1. The board, with both sides set up. Black plays down the board and White plays up.

Each player has eight pieces, called pearls. One side has the black pearls and the other the white. All pearls are alike and all move in the same way.

Black starts on rows 1 and 2, at the top of the diagrams, and plays down the board. White starts on rows 7 and 8, at the bottom, and plays up. These two rows at each end are shaded in the diagrams and are called the home rows. The heavy line between rows 4 and 5 is the center line. The board is not checkered and the squares have no other markings.

One word is needed before the rules. Two squares *touch* if they are next to each other in any direction, including diagonally. A square in the middle of the board touches eight others. A square on the edge touches five, and a corner square touches three. Moving and capturing are both defined by touching, and nothing in the game reaches further than one square.

3. The Object

A pearl *crosses* when it steps off the far edge of the board. A black pearl crosses from row 8. A white pearl crosses from row 1. Each crossing scores one point, and a pearl that has crossed is out of the game.

The player with more points at the end wins. So the game is at bottom a race: each side is trying to get its pearls to the far edge and off it. Everything else in the game, including capture, matters because of what it does to that race.

4. The Turn

Black moves first, and the players then take turns.

A turn is four steps. A *step* moves one pearl to an empty square that touches the square it stands on. The step can be forward, sideways, backward or diagonal. There is no jumping and no longer move.

The four steps can be divided in any way. A player can move one pearl four times, or four pearls once each, or anything in between. The order is the player's choice, and it often matters.

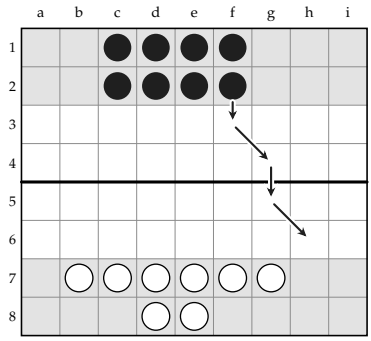

DIAGRAM 2. One pearl takes all four steps: f2–f3, f3–g4, g4–g5, g5–h6.

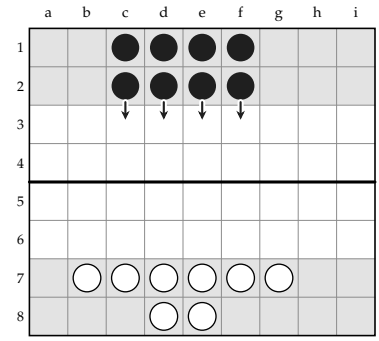

DIAGRAM 3. Four pearls take one step each: c2–c3, d2–d3, e2–e3, f2–f3.

A pearl on its last row crosses by taking one more step, off the board. That step counts as one of the four. A black pearl on row 5 therefore needs four steps to cross: three to reach row 8 and one to leave it.

All four steps must be taken. A player may not pass and may not stop after two or three steps if a legal step is still available. This rule looks like a detail. It decides a good many games, because a player who has no useful step must still take a harmful one.

The diagrams in this article all work the same way. Each shows the position before a turn. Arrows show the steps of that turn, and a cross through a pearl means that it was captured during the turn. The steps are also written under the diagram. A step is written as two squares, so e2–e3 means that the pearl on e2 steps to e3. When a step captures a pearl, the square of the captured pearl follows a cross, as in g6–f5 ×e4. A crossing is written e8–off. Beneath many diagrams is a small table of figures for each side, which Section 26 explains.

5. Capture

There is one rule of capture.

> *A pearl is captured when the enemy pearls touching it are at least twice as many as its own side on the spot. Its own side on the spot means the pearl itself and its supporting pearls, which are the pearls of its own side that touch it.*

The rule is easier to use than to state. A pearl with no supporting pearl counts as one, so two enemy pearls touching it capture it. A pearl with one supporting pearl counts as two, and four enemy pearls are needed to capture it. A pearl with two supporting pearls counts as three, and six are needed.

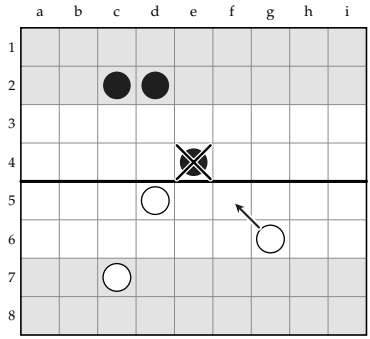

DIAGRAM 4. White's second pearl arrives beside e4: g6–f5 ×e4.

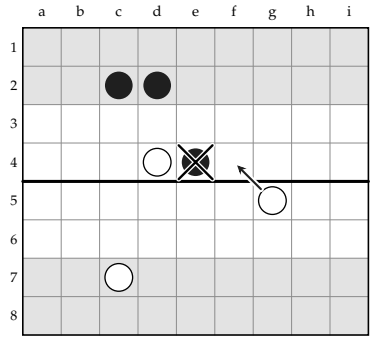

DIAGRAM 5. The two capturing pearls need not touch each other: g5–f4 ×e4.

In Diagram 4 the black pearl on e4 has no supporting pearl. One white pearl already touches it. When a second arrives, the count is two against

one, and e4 is captured. Diagram 5 shows that the capturing pearls can stand anywhere around the pearl. They do not have to be together.

Four more points complete the rule.

1. A captured pearl is removed at once and does not return.
2. The capturing pearls do not move. They stay on the squares they stood on.
3. A capture happens in the middle of a turn, at the moment a step makes the numbers come out. The player then goes on with whatever steps are left.
4. Captures score no points. Each side's captures are counted separately, and they are used only to settle a game that ends with the points level.

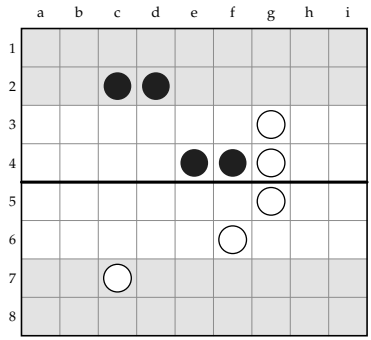

DIAGRAM 6. Three white pearls touch f4. It has a supporting pearl on e4, so four are needed. Nothing is captured.

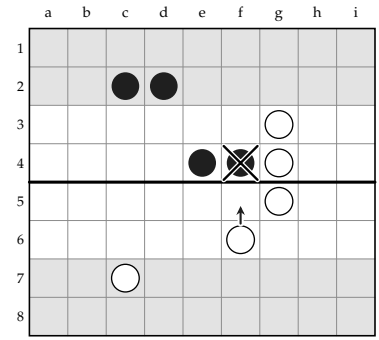

DIAGRAM 7. A fourth arrives and f4 is captured: f6–f5 ×f4. The pearl on e4 is touched by one white pearl and stays.

Diagrams 6 and 7 show the effect of one supporting pearl. The pearl on f4 has the pearl on e4 beside it. Three white pearls around it are not enough. A fourth is. Note that only f4 is taken. The capture rule is applied to each pearl by itself, and e4, with one white pearl touching it, is in no danger yet.

6. Two Things a Step May Not Do

A pearl may not step into capture. If a pearl would be captured on a square the moment it arrived, it may not step there. We call such a square *shut* to that pearl.

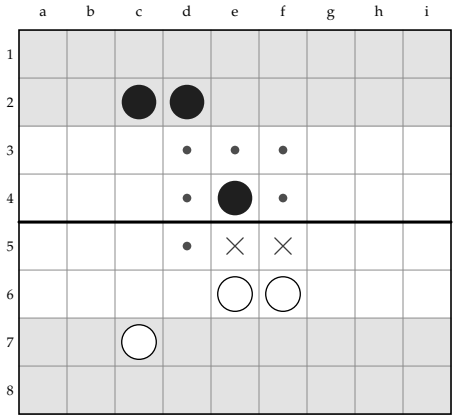

DIAGRAM 8. The pearl on e4 may step to any of the six dotted squares. The two squares marked with a cross are shut to it, because each touches two white pearls.

In Diagram 8 the squares e5 and f5 each touch both white pearls. A black pearl arriving on either with no supporting pearl beside it would be captured at once, so it may not go there. The square d5 touches only one white pearl and is open.

The rule protects only the pearl that is moving. A step is legal even if it causes another of the player's own pearls to be captured, for example by stepping away from a pearl that depended on it. This happens by accident in beginners' games and on purpose in experts' games. Sections 19 and 20 return to it.

A pearl may not end its turn on the square on which it began it. A pearl can wander during a turn, and may even pass back through its starting square, but it may not finish the turn there. A player therefore cannot use up steps by moving one pearl out and back.

When a pearl has no legal step, it does not move. If none of a player's pearls has a legal step, the turn ends early. This is rare, and Section 17 shows how it can happen.

7. How the Game Ends

> *The game ends at once when either player has no pearls left on the board. The player with more points wins. If the points are level, the player who has made more captures wins.*

A player's pearls leave the board in two ways, by crossing and by being captured. When the last one has gone, by either route, the game is over, even if the other player still has pearls on the board and steps left to take. A player may also resign at any time.

The rule does not say that the game goes on until all sixteen pearls have gone. It does not say that the player who empties the board first wins. And it does not say that the pearls still on the board at the end score anything. They score nothing. Each of these points has consequences, and the most striking are in Sections 19 and 20.

Those are all the rules.

Part Two
First Skills

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8. Setting Up

Each side places its eight pearls on its home rows. Usually both sides arrange their pearls as they like, adjust them after seeing the other side's arrangement, and begin when both agree to begin. This is called a gentlemen's game. If the players cannot agree, the game is a hostile game, and they place one pearl at a time, in turn. Black takes the first turn.

The home row nearer the center is the *front row*: row 2 for Black and row 7 for White. The other is the *back row*. A pearl on the front row starts one step nearer to crossing than a pearl on the back row.

The number of pearls on each row is governed by a short rule. White chooses how many pearls to put on its front row, anywhere from eight to none. Black may put on its front row at most two fewer than White

does, and may put fewer still if it wishes; the rest go on its back row. When White puts two or fewer forward, Black must place all eight on its back row. With Black taking the most the rule allows, there are nine arrangements, lettered A to I by White's number.

	White, front and back	*Black, front and back*
A	8 and 0	6 and 2
B	7 and 1	5 and 3
C	6 and 2	4 and 4
D	5 and 3	3 and 5
E	4 and 4	2 and 6
F	3 and 5	1 and 7
G	2 and 6	0 and 8
H	1 and 7	0 and 8
I	0 and 8	0 and 8

Within these numbers the pearls can stand on any files the player likes. In arrangements A to G, White starts two steps nearer to the far edge than Black, taking the eight pearls together. Black has the first turn in return.

Arrangements H and I make no sense for White. Black is all back in G, H and I alike, so moving a seventh or an eighth white pearl to the back row only adds steps to White's count against the same black setup. Good play starts from one of A to G, and the rest of this volume takes those seven as the arrangements of the game. Black can also hold back more pearls than the rule requires. Each one adds a step to Black's count. *The Level Lock*, in this volume, discusses when Black gains by holding pearls back.

For a first game we suggest arrangement C, set up as in Diagram 1. White puts six pearls in a row on b7 to g7 with two behind them on d8 and e8. Black puts four on c2 to f2 and four behind them on c1 to f1. Both shapes are compact, every pearl begins with supporting pearls beside it, and the race that follows is close. Section 23 shows how close.

There is a good deal more to be said about which files to choose and why. A new player does not need it yet. What matters at the start is that no pearl begins alone.

9. Using a Turn

Every step is one of three kinds.

A *forward* step takes a pearl one row nearer the far edge. It can be straight or diagonal. A diagonal step forward gains a row exactly as a straight one does, and changes file as well, without losing a step. This is how pearls go round things.

A *sideways* step stays on the same row. It gains nothing.

A *backward* step loses a row, which will have to be made up later.

The first habit to form is to count the forward steps in each turn before playing it. A turn of four forward steps is a full turn. A turn with a sideways step in it has given something away, and there should be a reason for it.

The second question is which pearls should take the steps. Four steps on one pearl carry it half the length of the board. Four steps on four pearls move a group up by one row. Both turns gain four rows in all, and neither is the faster. The difference is in what stands on the board afterwards. In the first case one pearl is far ahead and alone. In the second, four pearls are still together.

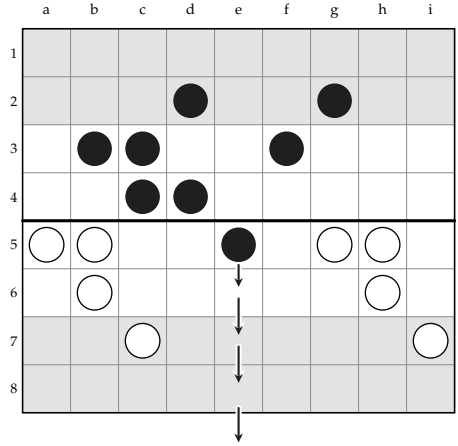


DIAGRAM 9. A pearl that starts the turn across the center line can cross in that turn: e5–e6, e6–e7, e7–e8, e8–off.

There is one case in which a single pearl should take everything. A pearl that has passed the center line is four steps or fewer from crossing. If its path is open, it can score in one turn, as in Diagram 9. The center line matters for this reason. A pearl beyond it is a threat to score on the very next turn, and the opponent has to reckon with it at once.

Count the forward steps in every turn. A pearl across the center line is one turn from scoring.

10. Strength in Numbers

The capture rule makes pearls harder to capture when they stand together. The table sets out the numbers.

The pearl has	*Enemy pearls needed to take it*
no supporting pearl	2
one supporting pearl	4
two supporting pearls	6
three supporting pearls	8

Two enemy pearls are easy to bring to one place in a turn. Four are hard. Six can be done only with most of a side's pearls and several turns of preparation. Eight cannot be done at all against a block of four, as Diagram 11 shows.

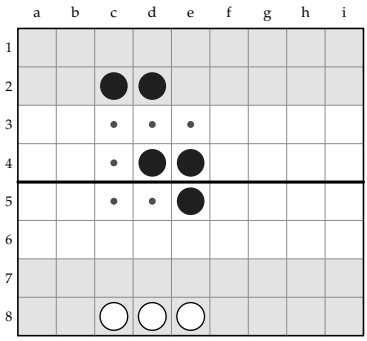


DIAGRAM 10. Three pearls that all touch one another. To take d4, White would need a pearl on each of the six dotted squares.

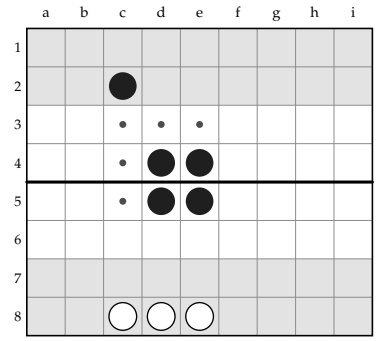


DIAGRAM 11. A block of four. Only five squares touch d4 from outside, and eight pearls would be needed. It cannot be taken.

A pearl supports another only if it touches it. Three pearls in a straight line are easier to capture than three in the shape of Diagram 10. Each end pearl of a line has one supporting pearl, while each pearl in Diagram 10 has two.

Supporting pearls have a second effect. A square that is shut to a lone pearl may be open to a pearl that has a supporting pearl beside the square.

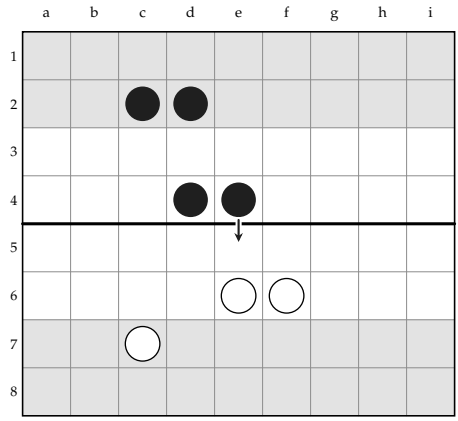


DIAGRAM 12. With a supporting pearl on d4, the pearl on e4 may step to e5. Two white pearls touch it there, and four would be needed.

Compare Diagram 12 with Diagram 8. The white pearls are the same. In the earlier diagram the black pearl was alone and e5 was shut to it. Here the pearl on d4 touches e5, so the pearl arriving there counts as two, and two white pearls are not enough to take it. A pair can go where a single pearl cannot.

A group loses no steps in the race by traveling together, since four pearls advancing one row each have made four forward steps.

> *Two enemy pearls capture a lone pearl. Four are needed against a pearl with one supporting pearl. Move in twos and threes.*

11. Seeing a Capture Coming

The opponent has four steps. That is further than it looks, and most pearls lost in early games are lost because their owner did not think four steps ahead.

In Diagram 13 Black has sent one pearl ahead to e5. No white pearl touches it, and it looks safe. It is not safe: the white pearl on d7 is one step from a square touching e5, and the pearl on g7 is one step from another. Two steps, and the pearl is gone, with half of White's turn still to play.

The check to make before finishing a turn is this. For each of your pearls that will stand with no supporting pearl, find the two enemy pearls

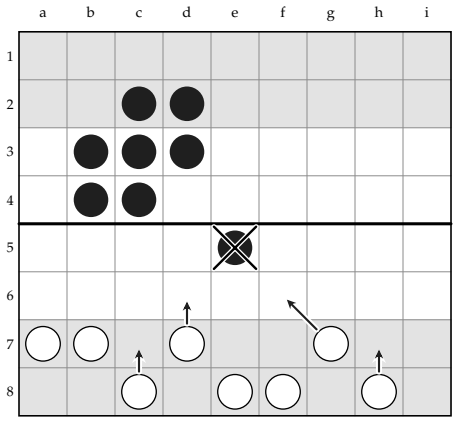


DIAGRAM 13. Black's pearl on e5 stands alone. White needs two steps to take it and has two to spare: d7–d6, g7–f6 ×e5, c8–c7, h8–h7.

nearest to it and count the steps each needs to reach a square that touches it. If the two numbers add up to four or less, expect to lose the pearl.

For a pearl with one supporting pearl the same check is made with four enemy pearls, and the answer is almost always that it is safe for now.

12. Making a Capture

To capture a lone pearl, bring two of your own to squares that touch it. Each of them must be able to stand where it arrives. The first to arrive touches only one enemy pearl and is usually safe. The second completes the capture.

To capture a pearl that has a supporting pearl, you would need four around it. In practice a pair is taken by first taking one of the two when it stands a little apart, or not taken at all.

Whether to capture is a harder question than how. Three things should be weighed every time.

How many steps will it lose? Count the steps of the capture that do not go forward. A side that captures with two forward steps loses no step. A side whose capture needs a sideways step and a backward step loses three.

What does it leave? After the capture your two pearls are standing where the enemy pearl was, perhaps apart from the rest of your side. Make the check of Section 11 on them.

How far had the pearl to go? A pearl one step from crossing was about to score a point. Taking it stops that point. A pearl still on its home rows was

many turns from scoring. Taking it helps the capturing side less than it seems, and sometimes it helps the side that lost the pearl. The reason is in Part Four, and the game in Part Six turns on it.

Take runners. Think twice about taking a pearl that has hardly moved.

13. A Plan for the First Games

A new player needs something to do from the first turn. The following plan is simple, and it is sound as far as it goes.

At the start. Play four forward steps every turn. Keep every pearl touching a supporting pearl. Do not send a pearl across the center line by itself unless it can cross the board on the following turn and nothing can reach it first.

When the fronts meet. The two sides usually come into contact along the center line with every pearl supported, and nothing can be captured. Do not break up a group to attack one. Look for the side of the board where the opponent has fewer pearls, and go round that way with diagonal steps, two or three pearls together.

When the sides have passed. Count, by the method of Part Four. If you are ahead, run, and keep your pearls away from the opponent's slow pearls. If you are behind, something has to change. Look for a runner that can be taken from behind, a pearl standing alone, or a slow pearl of your own that is holding you back.

At the end. Count every turn. Cross a pearl when it can cross, since a point scored cannot be taken away and a pearl on the last row can. And keep in mind that the game stops when either side has no pearls left.

Part Three
Surprises

The rules of Part One are few, but they combine in ways that a new player does not foresee. This Part sets out eight of them. Each follows strictly from the rules, and each will turn up in the reader's own games sooner than expected.

14. One Step, Two Pearls

A capture changes the numbers for every pearl nearby, and the rule is applied again at once. So one step can take more than one pearl.

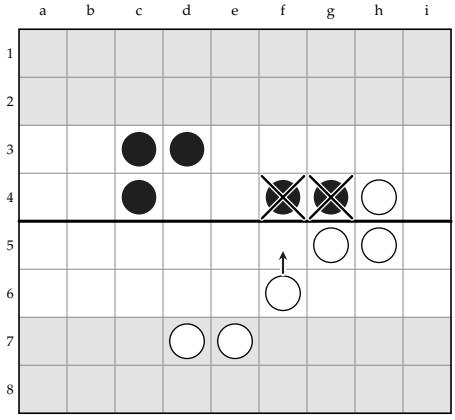


DIAGRAM 14. One step takes both pearls: f6–f5 ×g4, f4.

In Diagram 14 the black pearls on f4 and g4 protect each other. Three white pearls touch g4, and four are needed. White steps to f5. That is the fourth pearl against g4, which is captured. The pearl on f4 has now lost its supporting pearl. Two white pearls touch it, on f5 and g5, and that is enough. It is captured as well, by the same step.

A pair is as strong as its weaker pearl. When one goes, the other is a lone pearl standing among the pearls that took the first.

15. Two Pearls Block Nothing

New players try to stop an advancing pearl by standing a pearl or two in its way. It does not work.

Two black pearls side by side shut four squares to a lone white pearl: the two in front of them and the two behind. Every other square is open. In Diagram 15 the white pearl changes file with its first step, which is a forward step and loses nothing, and walks past on the f-file. It touches one black pearl on the way, which is harmless.

A single pearl standing in the way does even less, since by itself it shuts no square at all. Three in a row are little better: a runner that begins to change file a row earlier passes them too without losing a step. A side cannot stop a runner with a pearl or two in front of it. It needs the arrangement in the next section.

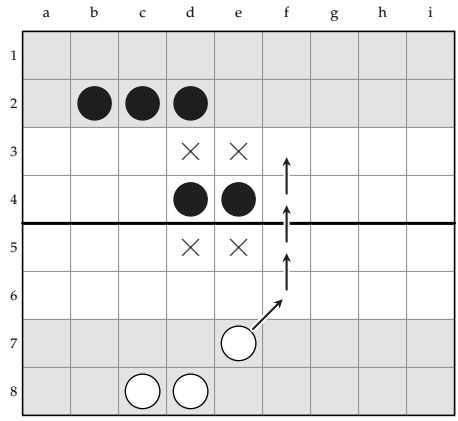


DIAGRAM 15. Two pearls side by side shut the four marked squares to a lone pearl. White's runner goes by without losing a step: e7–f6, f6–f5, f5–f4, f4–f3.

16. Five Pearls Close the Board

Now stand five pearls on alternate files, all on the same row.

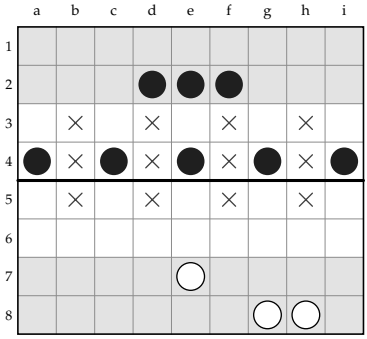


DIAGRAM 16. Five pearls on alternate files. Every marked square is shut to a lone white pearl, and no lone pearl can get past.

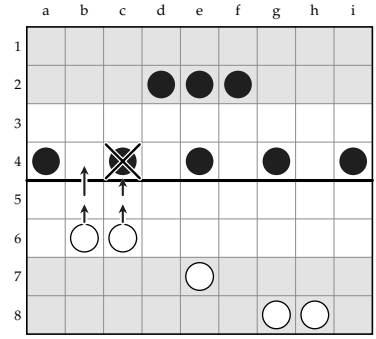


DIAGRAM 17. Two pearls together open it in two steps and are through in four: b6–b5, c6–c5 ×c4, b5–b4, c5–c4.

In Diagram 16 each gap between two black pearls is touched by both of them. So is the square in front of each gap and the square behind it. A lone white pearl can come up to the line, and can even stand directly in front of a black pearl, but it cannot step into a gap or past one. With five pearls Black has closed all nine files, and has three pearls left over.

No other arrangement closes so much of the board with so few pearls, and no other is so easily broken. Diagram 17 shows how. Every pearl in the line stands alone. Two white pearls arriving together take one of them in two steps: the first steps in front of a gap, which it may do because its supporting pearl is beside it, and the second steps in front of the black pearl. The pearl on c4 is captured, and both white pearls walk through the hole.

So a line of this kind stops single pearls completely and does not stop pairs. Whether to build one, and when, depends on what the opponent has coming.

17. The Pearl That Cannot Move

A pearl can be left with no legal step without being captured.

In Diagram 18 one white pearl touches the black pearl on i4, which does it no harm. Each of the four empty squares around it touches two white pearls, and the pearl may not step into capture. It stays where it is until something changes. Three pearls hold it. On the edge of the board a pearl has fewer squares to go to, and fewer pearls are needed to shut them all.

A pearl held like this has not been captured and still counts as a pearl on the board. That can matter a great deal at the end of a game, as Part Four explains. And if a player's only remaining pearls are all held, that player must sit and watch, because a turn with no legal step is no turn at all.

18. The Capture from Behind

Pearls capture in every direction, including backward. A runner that has slipped past the opposing pearls is not yet safe.

In Diagram 19 a white pearl has got round Black's pearls and stands on f3. It needs three steps to cross and will take them next turn. One black pearl, on e4, already touches it. Black steps the pearl on d4 back to e3, and the runner is captured.

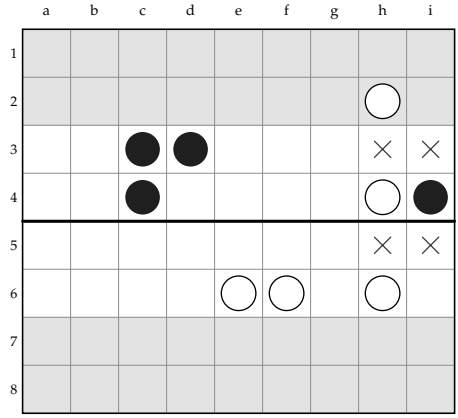


DIAGRAM 18. The pearl on i4 has no legal step. Each square it could go to touches two white pearls.

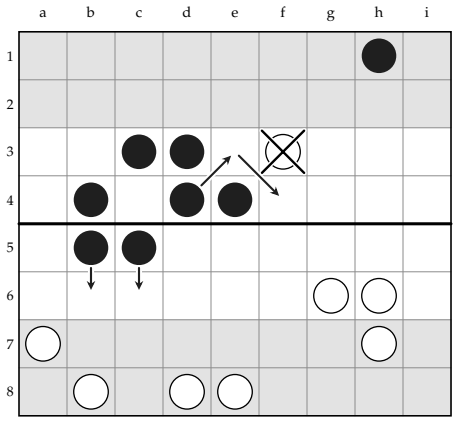


DIAGRAM 19. A step backward completes the capture: d4–e3 ×f3, e3–f4, b5–b6, c5–c6.

The step to e3 was a backward step, and the pearl then steps forward again to f4. It may not end the turn back on d4, where it began. So Black lost two steps in making the capture: one to go back and one to recover the row. Two steps are few to lose in order to take a pearl that was about to score.

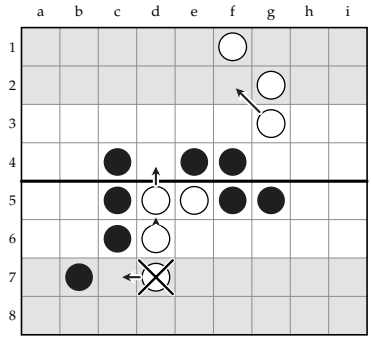
The position also shows a general point. A pearl that goes ahead alone has enemies behind it as well as in front, and the ones behind are often nearer.

19. Six Captures and a Lost Game

The game ends when either side has no pearls on the board, and it is decided by points. A player who forgets this can win every fight and lose the game. The three diagrams that follow are three consecutive turns from a game.

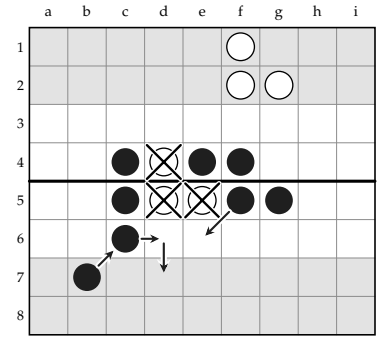
In Diagram 20 Black has all eight pearls and has already made one capture. White has seven pearls. Three of them are close to crossing, on f1, g2 and g3. The other four are far behind, in the middle of Black's pearls, with a long way to go.

White's turn looks like a blunder. It steps a pearl from d7 to c7, next to two black pearls. The step is legal because the pearl on d6 is beside it. Then White moves the pearl on d6 away. The pearl on c7 is left alone between two black pearls and is captured. White has given it away, and Black has not had to take a step.



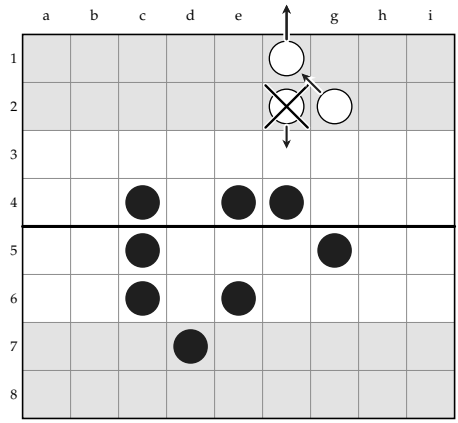
	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	1	0	2	0
Σ	27:7:1	29:8:3	22:6:2	19:5:1
∨	5	7	5	5

DIAGRAM 20. White's turn: g3–f2, d5–d4, d7–c7, d6–d5 ×c7. White has given a pearl away.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	2	0	5	0
Σ	22:6:2	19:5:1	8:2:0	5:2:3
∨	5	5	3	2

DIAGRAM 21. Black's turn: c6–d6, f5–e6, b7–c6 ×d5, e5, d4, d6–d7. Three pearls with one step.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	2
x	5	0	6	0
Σ	8:2:0	5:2:3	5:2:3	0:0:0
∨	3	2	3	–

DIAGRAM 22. White's turn: f1–off, f2–f3, g2–f1 ×f3, f1–off. White has no pearls left and the game is over. White wins by two points to none.

Black replies with one of the most satisfying turns the game offers. The step from b7 to c6 captures three white pearls at once. The pearl on d5

goes first, then the two that depended on it. Black has now made five captures and has not lost a pearl.

White's last turn is Diagram 22. The pearl on f1 crosses. The pearl on f2 steps up to f3, beside two black pearls, and the pearl on g2 steps away from it and crosses in its turn. The pearl on f3 is captured. White has no pearls on the board, so the game ends at once. White has two points and Black has none.

Black made six captures, lost no pearl, and did not score a point. Look at what each side needed. Once White was down to a few pearls, White had only to get those few across. Black needed just as many crossings to draw level on points, and Black's pearls were all still in the middle of the board. Every capture Black made removed a slow white pearl that White would otherwise have had to bring home, and brought the end of the game nearer.

Results like this are not freaks. A side can lose seven pearls and win by one point to none, if its last pearl crosses before the opponent has scored. The lesson for the new player is to keep one question in mind through every fight: if the game ended soon, who would have the points?

20. Giving a Pearl Away

The last section showed a player losing pearls on purpose. It can be done in two ways, and both are legal.

Walking away. A pearl that stands next to enemy pearls is safe while a supporting pearl touches it. If that pearl steps away, the pearl may be captured. The rule against stepping into capture applies only to the pearl that moves, and the pearl that moved is not the one captured.

Stepping in and walking away. A pearl may step next to two enemy pearls if a supporting pearl is beside the square. The supporting pearl then leaves. White did this in Diagram 20, with the steps d7–c7 and d6–d5.

Why would a player want to lose a pearl? Because a pearl far from crossing can keep its side from finishing. If the game is going to be decided by the pearls that are nearly home, a pearl many steps behind them is only something that has to be waited for, and the side that owns it would be better off without it. The opponent can decline to capture such a pearl. The opponent cannot decline when the owner arranges the capture itself.

It follows that standing two pearls next to an enemy straggler is not always safe, even when nothing can capture them. They may be the two pearls the opponent needs in order to be rid of it.

21. The Standoff

When both sides advance in a line, the lines meet at the center and stop.

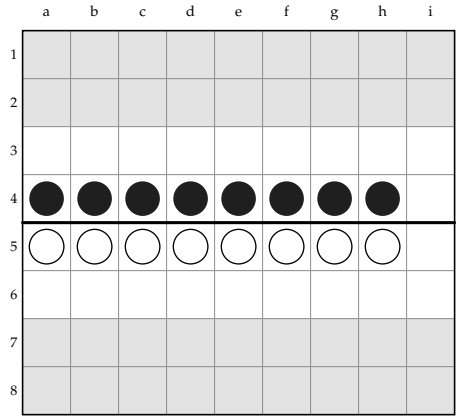


DIAGRAM 23. Two full lines. No pearl on either side can step forward except round the end, on the i-file.

In Diagram 23 every square in front of a pearl is occupied. Neither side can advance, except by sending a pearl round the end of the line. Each side must take four steps every turn, so the players step back and across behind their own lines, changing nothing, and each waits for the other to do something.

A pearl may not return to the square it started the turn on, but two pearls may change places. One steps back, the pearl beside it steps across into its square, and the first goes round into the square the second pearl left. That takes four steps and leaves the position exactly as it was. So a side can mark time. In ordinary play nothing forbids this. The full hostile game limits it: when both sides have marked time for two turns running, the side that did so first must change its position. This article and *The Level Lock* assume that rule. The side that has to open the position first often loses steps or pearls in doing so.

Positions of this kind have their own logic, and whole games are decided by who runs out of harmless steps first. They are the subject of *The Level Lock*, in this volume. For now it is enough to know that the standoff exists, that it is not a draw, and that the rule requiring four steps is what breaks it.

Part Four
Counting

Players who do not count judge a position by how it looks. Players who count know. The method in this Part needs only addition, and it is the single most useful habit a new player can form.

22. The Count

A pearl's *distance* is the number of steps it needs in order to cross. A pearl on its last row needs one step, the step off the board. A pearl one row further back needs two, and so on. A black pearl on row 2 needs seven. A white pearl on row 7 also needs seven.

A side's *count* is the sum of the distances of its pearls. It is the number of forward steps that side must still make.

In Diagram 24 White has six pearls at seven steps and two at eight, which is 58. Black has four at seven and four at eight, which is 60. This is the two-step start that White receives in every arrangement from A to G.

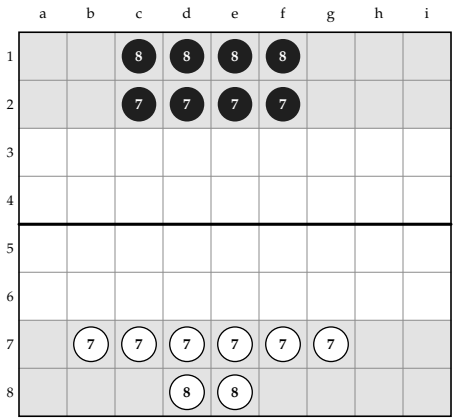


DIAGRAM 24. The starting position of Diagram 1 with each pearl's distance marked. White's count is 58 and Black's is 60.

23. From the Count to Turns

A turn is four steps, so a count is turned into turns by dividing by four and rounding up. White's 58 is fifteen turns: fourteen turns make 56 steps and the last two steps need a fifteenth. Black's 60 is fifteen turns exactly.

Both sides need fifteen turns. Black moves first, so Black's fifteenth turn comes before White's. If both sides simply ran, and neither lost a step, Black would finish first and win.

> *Fewer turns is ahead. With equal turns, the side that moves first is ahead.*

Now look at how narrow that is. Black's 60 is fifteen turns with nothing to spare. One lost step makes it 61, which is sixteen turns, and White would then finish first. White's 58 has two steps to spare, since 59 and 60 are still fifteen turns. So Black is ahead and loses the lead if it loses a single step, while White is behind and can lose two steps in making trouble without needing another turn. Something like this is true at the start of every game.

A step is *lost* when it does not go forward. A sideways step is one step lost. A backward step is two steps lost, the step itself and the step needed to make it up.

The same sum can be done for every arrangement.

	Count		*Turns*		
	White	*Black*	*White*	*Black*	*Ahead*
A	56	58	14	15	White
B	57	59	15	15	Black
C	58	60	15	15	Black
D	59	61	15	16	White
E	60	62	15	16	White
F	61	63	16	16	Black
G	62	64	16	16	Black
H	63	64	16	16	Black
I	64	64	16	16	Black

The table shows who is ahead before a step has been played. It does not show who will win. The count is a statement about a race in which nobody interferes, and the whole of the play consists of interfering: making the opponent lose steps, and not losing one's own. A player who knows the count knows what has to be achieved. The side ahead needs only to keep things as they are. The side behind has to change something.

24. When Pearls Have Been Captured

Captures change the count in a way that is not obvious, and that explains the strange results of Part Three.

In Diagram 25 Black has six pearls left and White has all eight. Neither side has scored.

Black can score six points at most. Suppose White gets six pearls across. Then the best Black can do is to get all six of its own across as well, and the points are level at six each. Level points are settled by captures, and White has made two to Black's none. So six crossings are enough for White. It does not need eight.

Black, for its part, wins if all six of its pearls cross before White's sixth does, because the game ends when Black's last pearl leaves the board, and White will then have fewer than six points.

So both sides are racing to six. We call this number the *target*. It is found by taking, for each side, its points plus its pearls still on the board, and choosing the smaller of the two results.

> *The target is the smaller of the two sides' totals of points and pearls. Each side counts only the steps it needs to reach the target.*

White needs six crossings and will use its six nearest pearls. Their distances are 2, 2, 3, 3, 5 and 5, which is 20, or five turns. The pearls on g7 and h8 are not counted, because White does not need them. Black needs every pearl it has. Its distances are 2, 2, 4, 4, 6 and 7, which is 25, or seven turns. White is well ahead, although Black's two front pearls are the nearest to crossing on the board.

Three things follow.

A capture lowers the target. When a pearl is taken from the side whose total of points and pearls is the smaller, or from either side when the totals are equal, there is one crossing fewer to race for.

Both counts fall. The side that lost the pearl no longer has to bring it home, so its count falls by that pearl's distance. The side that made the capture needs one crossing fewer, so its slowest counted pearl drops out, and its count falls by that pearl's distance. A capture helps the side whose count falls further. If the pearl taken was far from home, and the capturing side's slowest counted pearl was nearer, the capture has helped the side that lost the pearl.

The side with the smaller total needs every pearl it has. A pearl that is held, as in Section 17, or simply left far behind, still has to cross if its side is to reach the target. This is why a side sometimes gives such a pearl away, as

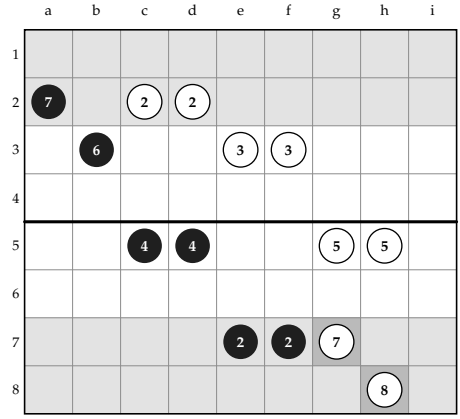


DIAGRAM 25. Black has lost two pearls. White needs only six crossings, and its two slowest pearls, on the shaded squares, no longer count.

in Sections 19 and 20. Once it is captured, the target falls and the pearl is no longer needed.

There is more to this, including the case in which the side that loses the pearl is the side ahead in captures. The full treatment is in *Arithmetic Margrites*, in this volume. What is set out here is enough to follow the game in Part Six.

25. Counting at the Board

The whole method is five short steps.

1. For each side, add its points to its pearls on the board. The smaller total is the target.
2. For each side, subtract its points from the target. That is the number of crossings it still needs.
3. For each side, add up the distances of that many of its nearest pearls. That is its count.
4. Divide each count by four and round up. That is the number of turns each side needs.
5. The side that needs fewer turns is ahead. If they need the same number, the side whose turn it is to move is ahead.

With practice this takes half a minute. It should be done every few turns, and always before making a capture or allowing one.

26. The Table Beneath a Diagram

Beneath most diagrams in this volume is a small table of figures. It sets out the count for both sides, so that the reader does not have to work it out for every diagram.

The table has a column for Black, marked •, and a column for White, marked ○. Its four rows are these.

1. The row marked + gives the side's points.
2. The row marked *x* gives the captures the side has made.
3. The row marked Σ gives the side's count to the target, followed by the turns that count needs and the spare steps in its last turn. So 16:4:0 is a count of sixteen, which is four turns with no step to spare, and 13:4:3 is a count of thirteen, which is four turns with three steps to spare. The sign Σ is the mathematical sign for a sum, and the count is the sum of the distances of the pearls a side needs.
4. The row marked ∨ gives the distance of the side's slowest counted pearl. This is the pearl that drops off the count if the target falls, as Section 24 explains.

The figures are for the position shown on the board. Where a diagram shows a turn, the table has a second pair of columns, headed *after*, with the figures once the turn has been played.

Each figure changes for a reason that can be found on the board.

Points. The figure in the + row rises by one each time a pearl of that side crosses.

Captures. The figure in the *x* row rises by one each time the side captures a pearl, whether its own step made the capture or the opponent gave the pearl up.

The count. A forward step takes one off the side's count. A sideways step leaves it as it was. A backward step adds one. A capture changes both counts at once. The side that loses the pearl no longer has to bring it home, so its count falls by that pearl's distance. If the capture lowers the target, the other side needs one crossing fewer, and its count falls by the distance of the pearl it no longer needs, which is the pearl in its ∨ row. The turns and the spare steps follow from the count by dividing by four, and a single lost step can change the turns.

The slowest counted pearl. The figure in the ∨ row falls when that pearl steps forward. It also changes when a capture changes which pearls are counted. A pearl that stays where it is can drop off its side's count, and then the side's ∨ figure falls to the distance of the next slowest pearl.

The table beneath Diagram 39, in the game of Part Six, shows all of these at once.

	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	1	0
Σ	36:9:0	34:9:2	26:7:2	29:8:3
∨	6	7	5	7

Before Black's seventh turn the figures are 36:9:0 for Black and 34:9:2 for White, and both sides need nine turns. Black plays four forward steps, which take four off its count, and captures the white pearl on d5. White no longer has to bring d5 home, so White's count falls by its distance, five, to 29:8:3. The capture lowers the target from eight to seven. Black needs one crossing fewer, and its slowest counted pearl, on d3, comes off the count. That pearl was six steps from crossing, which was the figure in Black's ∨ row, so Black's count falls by six more, to 26:7:2. Black's ∨ figure falls to five, the distance of its next slowest pearl, and Black's *x* figure rises to one.

Part Five
Ten Mistakes

27. The Errors That Lose Early Games

One: sending a pearl ahead alone. Four steps on one pearl feel like progress. The pearl arrives among the enemy with no supporting pearl, and two enemy pearls remove it. Make the check of Section 11 first.

Two: stepping away from a supporting pearl. The opposite mistake. A pearl that is safe because a supporting pearl touches it is lost when that pearl moves on.

In Diagram 26 two white pearls touch the black pearl on d5. It is safe because c4 touches it too. Black steps c4 forward to b5, which no longer

touches d5, and the pearl is captured by Black's own step. Before moving a pearl, look at what it was holding up.

Three: sideways steps without a reason. Each one is a step lost. The side that is ahead on the count usually has none to lose.

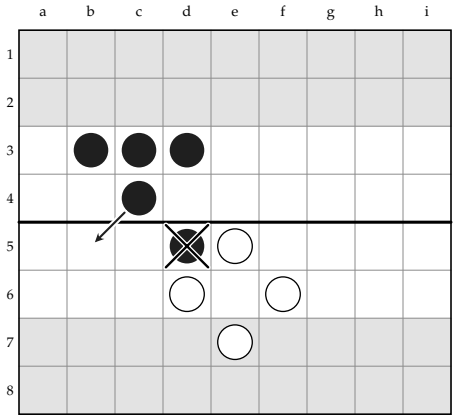


DIAGRAM 26. A natural forward step loses a pearl: c4–b5 ×d5.

Four: taking a pearl because it can be taken. Ask the three questions of Section 12. Above all, ask how far the pearl had to go. Taking a runner is nearly always right. Taking a straggler often helps its owner.

Five: forgetting that the opponent has four steps. A threat that needs four steps to carry out is a threat for the next turn. Look at the whole board before finishing a turn, not only at the pearls that have just moved.

Six: blocking with one or two pearls. They block nothing, as Section 15 showed. The steps used to place them are lost, and the runner goes by.

Seven: chasing what cannot be caught. Count before sending pearls back after a runner. If the runner needs fewer steps to cross than the chasers need to reach it, the chase loses twice: the runner scores, and the chasers have gone backward at two lost steps for every step.

Eight: standing beside a straggler. Two pearls that touch an enemy pearl far from home give its owner the means to be rid of it. The game in Part Six turns on this.

Nine: forgetting how the game ends. The game stops the moment either side has no pearls on the board. Before a step that crosses your own last pearl, or captures the opponent's last pearl, check the points. A player who is behind on points and removes the last pearl of either color has ended the game and lost it.

Ten: not counting. Every other mistake on this list is easier to see with the count in hand.

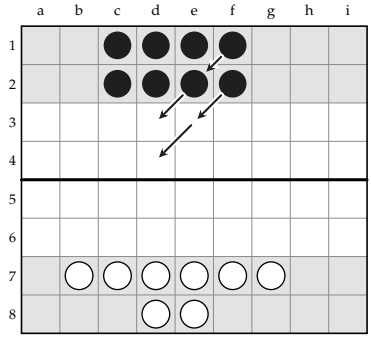
Part Six
A Game

28. The Game

The game that follows was played from the setup of Diagram 1. It lasted twelve turns a side. White won by six points to five, and the margin at the end was a single step.

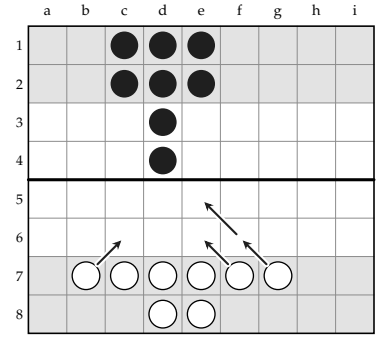
Both sides advance in groups. The fronts meet, touch all along the line, and pass. On the seventh turn Black makes a capture that needs six pearls, and White answers by giving a pearl away. From then on the count decides the game, five turns before the last pearl crosses.

Each diagram shows the position before a turn, with the steps of that turn marked. The counts given in the notes are the counts after the turn has been played.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	60:15:0	58:15:2	56:14:0	58:15:2
∨	8	8	8	8

DIAGRAM 27. Black 1. e2–d3, f1–e2, f2–e3, e3–d4.

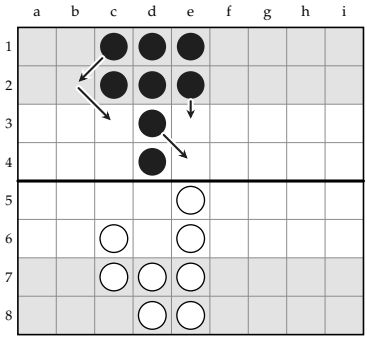


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	56:14:0	58:15:2	56:14:0	54:14:2
∨	8	8	8	8

DIAGRAM 28. White 1. b7–c6, f7–e6, g7–f6, f6–e5.

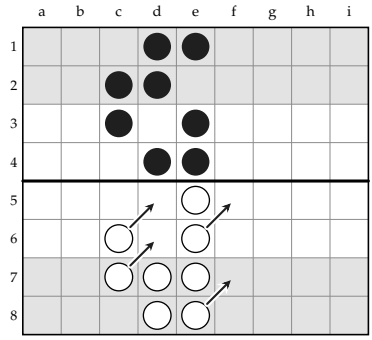
Black 1. Four forward steps, and Black's count falls from 60 to 56. The front pearl from f2 takes two of them and ends on d4. Black is drawing its block in toward the d-file, and every black pearl still has a supporting pearl.

White 1. White answers in kind: four forward steps, 58 to 54. The wing pearls come in toward the middle, and the pearl on e5 has reached the center line with e6 behind it.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	56:14:0	54:14:2	52:13:0	54:14:2
∨	8	8	8	8

DIAGRAM 29. Black 2. c1–b2, b2–c3, e2–e3, d3–e4.

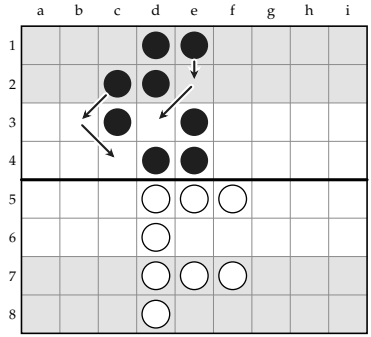


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	52:13:0	54:14:2	52:13:0	50:13:2
∨	8	8	8	8

DIAGRAM 30. White 2. c6–d5, e6–f5, c7–d6, e8–f7.

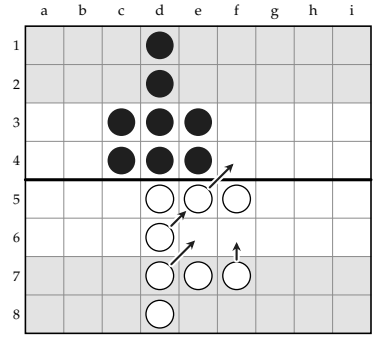
Black 2. A back pearl takes two steps to c3, and a second pearl arrives on the fourth row. The pearl on e4 now touches White's pearl on e5. Neither can be taken. Each has a supporting pearl beside it, and four enemy pearls are needed to capture a pearl with even one supporting pearl. Count 52.

White 2. White lines up three pearls on d5, e5 and f5, directly against Black's two. Three white pearls now touch e4, which would be more than enough against a lone pearl. The pearl on e4 has d4 and e3 beside it, so it counts as three, and six would be needed. Count 50.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	52:13:0	50:13:2	48:12:0	50:13:2
∨	8	8	8	8

DIAGRAM 31. Black 3. e1–e2, c2–b3, e2–d3, b3–c4.

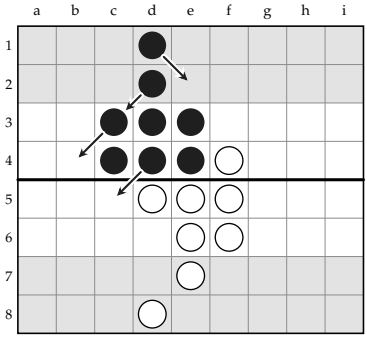


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	48:12:0	50:13:2	48:12:0	46:12:2
∨	8	8	8	8

DIAGRAM 32. White 3. e5–f4, d6–e5, d7–e6, f7–f6.

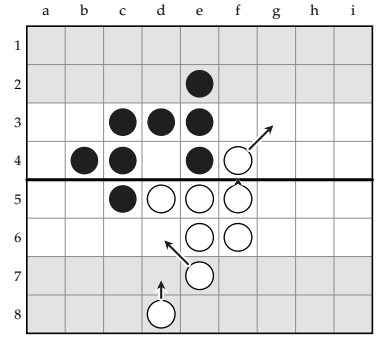
Black 3. Black fills in behind. Six black pearls now stand in a block on rows 3 and 4, three wide and two deep, and nothing in it can be taken. Count 48.

White 3. The pearl on e5 steps to f4, across the center line and against the side of the black block, and the pearls behind it close up. It touches e4 and e3, but it has e5 and f5 beside it. The two fronts are in contact all along the line and neither side can capture anything. This is Section 10 in practice. Count 46.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	48:12:0	46:12:2	44:11:0	46:12:2
∨	8	8	7	8

DIAGRAM 33. Black 4. d1–e2, c3–b4, d2–c3, d4–c5.

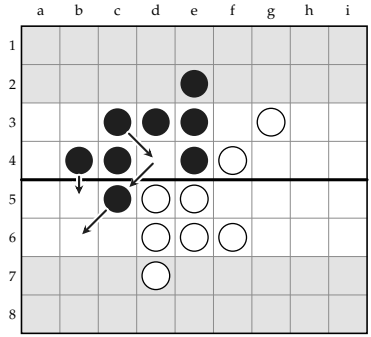


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	44:11:0	46:12:2	44:11:0	42:11:2
∨	7	8	7	7

DIAGRAM 34. White 4. f4–g3, f5–f4, e7–d6, d8–d7.

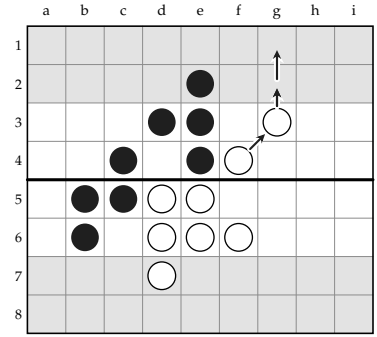
Black 4. There is no way through the middle, so Black goes round. The pearl on d4 steps to c5, beside the end of White's line, another goes to b4, and the back pearls come up behind them. Every step is a diagonal step forward, and so nothing is lost by changing direction. The pearl left on e4 is touched by four white pearls. It has two supporting pearls, so six would be needed. Count 44.

White 4. White goes round on the other side. The pearl on f4 steps on to g3, another takes its place, and the two back pearls move up. Count 42.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	44:11:0	42:11:2	40:10:0	42:11:2
∨	7	7	7	7

DIAGRAM 35. Black 5. c3–d4, b4–b5, c5–b6, d4–c5.

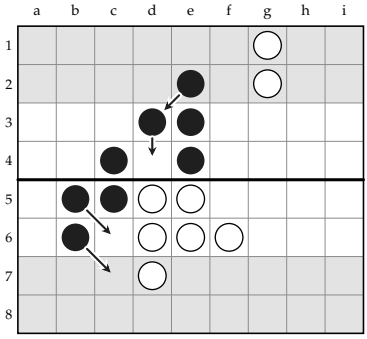


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	40:10:0	42:11:2	40:10:0	38:10:2
∨	7	7	7	7

DIAGRAM 36. White 5. g3–g2, g2–g1, f4–g3, g3–g2.

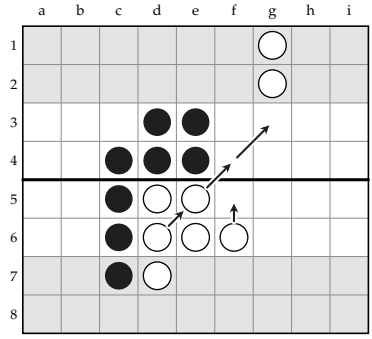
Black 5. Black's leading group goes up the b- and c-files, to b5, c5 and b6. Four forward steps again. Count 40.

White 5. White runs. Two pearls go down the g-file to g1 and g2, one step and two steps from crossing. They touch each other, and Black has one pearl anywhere near them, so they cannot be caught. Count 38. Each side needs ten turns and Black is to move. Black is ahead, by exactly as little as at the start.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	40:10:0	38:10:2	36:9:0	38:10:2
∨	7	7	6	7

DIAGRAM 37. Black 6. d3–d4, e2–d3, b5–c6, b6–c7.



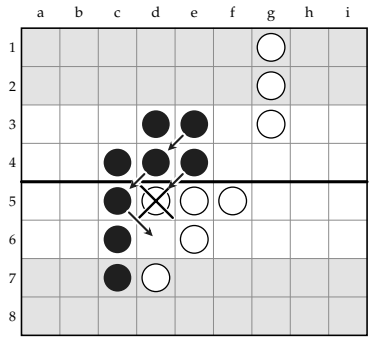
	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	36:9:0	38:10:2	36:9:0	34:9:2
∨	6	7	6	7

DIAGRAM 38. White 6. f6–f5, e5–f4, f4–g3, d6–e5.

Black 6. Four more forward steps. The leading pearls reach c6 and c7 and the middle closes up behind them. Count 36, which is nine turns with nothing to spare. The pearls on c6 and c7 now both touch White's back pearl on d7, and the game turns on that.

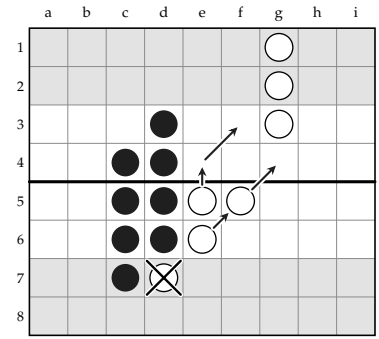
White 6. White brings its middle pearls on. Count 34. Both sides need nine turns and Black is to move, so Black is still ahead.

One white pearl is doing two jobs. The pearl on e6 is the only supporting pearl of the back pearl on d7. It is also one of the two supporting pearls of the pearl on d5, which now has five black pearls round it and would be captured if it lost either of them. So White cannot yet move e6 away.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	1	0
Σ	36:9:0	34:9:2	26:7:2	29:8:3
∨	6	7	5	7

DIAGRAM 39. Black 7. c5–d6, d4–c5, e3–d4 ×d5, e4–d5.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	1	0	2	0
Σ	26:7:2	29:8:3	21:6:3	18:5:2
∨	5	7	5	5

DIAGRAM 40. White 7. e5–e4, e4–f3, f5–g4, e6–f5 ×d7.

Black 7. The capture. White's pearl on d5 has two supporting pearls, on e5 and e6, so six black pearls are needed, and Black finds them. After the first two steps there are five round it, on c4, c5, c6, d6 and e4. The third step, e3–d4, is the sixth. The fourth step puts a pearl on the empty square. It is a fine turn to play, and every step in it is forward.

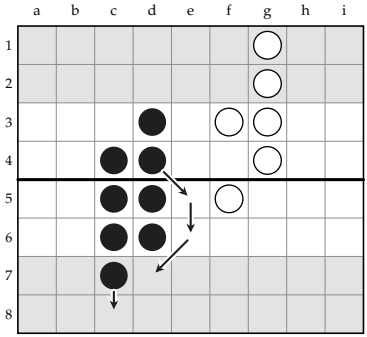
It also changes the target, which is now seven. White no longer has to bring d5 home, which takes five from White's count. Black needs one crossing fewer, and its slowest pearl, on d3, drops out, which takes six from Black's. Black 26, White 29. On the count Black has gained a step by the capture.

It is nevertheless the losing turn. The pearl on e6 was tied to d5, and d5 is gone. And after the capture three black pearls, on c6, c7 and d6, touch the pearl on d7. Black had another turn available. Four diagonal steps forward, c7–b8, c6–b7, c5–b6 and c4–b5, would have taken every black pearl away from d7 and lost nothing. White's slow pearl would have stayed on White's count, and Black would have kept its lead.

White 7. White moves the pearl on e6 away. The pearl on d7 is left alone against three black pearls and is captured, by White's own step. It had seven steps to go and was shut in behind Black's pearls. White needed it home, and now does not.

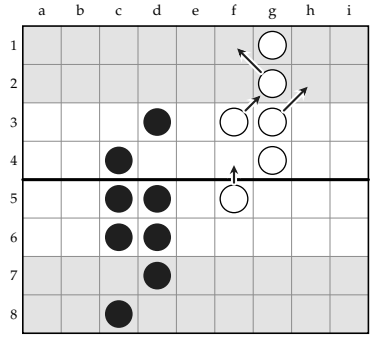
The target is six. White's count falls by the four forward steps of the turn and by the seven that d7 no longer has to make: 29 to 18. Black drops its next slowest pearl, on c4, which takes five from its count: 26 to 21. White needs five turns. Black needs six. Black moves first, but White's fifth

turn comes before Black's sixth. The lead has changed hands, and it does not change back.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	2	0	2	0
Σ	21:6:3	18:5:2	17:5:3	18:5:2
∨	5	5	4	5

DIAGRAM 41. Black 8. d4–e5, e5–e6, e6–d7, c7–c8.

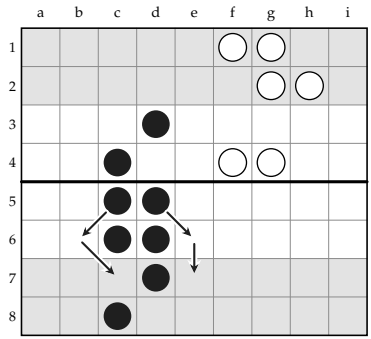


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	2	0	2	0
Σ	17:5:3	18:5:2	17:5:3	14:4:2
∨	4	5	4	4

DIAGRAM 42. White 8. g2–f1, f3–g2, g3–h2, f5–f4.

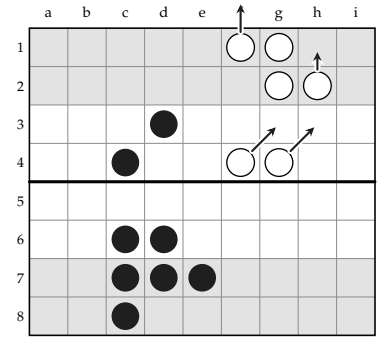
Black 8. Black sees what has happened and runs. Four forward steps with its nearest pearls. Count 17.

White 8. White runs too. Count 14. The two sides have passed each other. There is nothing left for either to capture and nothing to block, and from here the game is the count and nothing else.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	2	0	2	0
Σ	17:5:3	14:4:2	13:4:3	14:4:2
∨	4	4	3	4

DIAGRAM 43. Black 9. c5–b6, d5–e6, b6–c7, e6–e7.

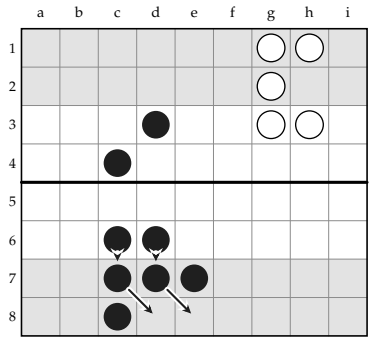


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	1
x	2	0	2	0
Σ	13:4:3	14:4:2	13:4:3	10:3:2
∨	3	4	3	3

DIAGRAM 44. White 9. f1–off, h2–h1, f4–g3, g4–h3.

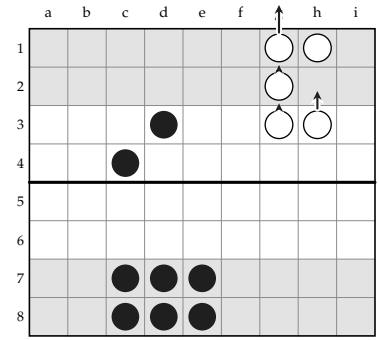
Black 9. Count 13: four turns, with three steps to spare that are of no use.

White 9. The pearl on f1 crosses and White has the first point. Count 10.



	BEFORE		AFTER	
	●	○	●	○
+	0	1	0	1
x	2	0	2	0
Σ	13:4:3	10:3:2	9:3:3	10:3:2
∨	3	3	2	3

DIAGRAM 45. Black 10. c7–d8, c6–c7, d7–e8, d6–d7.

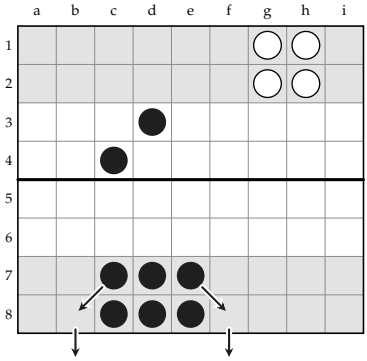


	BEFORE		AFTER	
	●	○	●	○
+	0	1	0	2
x	2	0	2	0
Σ	9:3:3	10:3:2	9:3:3	6:2:2
∨	2	3	2	2

DIAGRAM 46. White 10. g1–off, g2–g1, g3–g2, h3–h2.

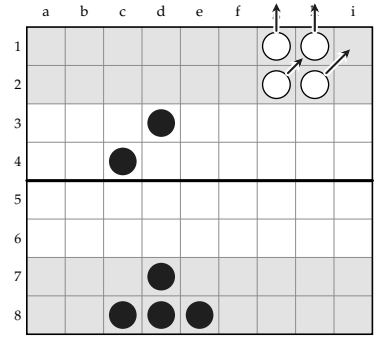
Black 10. Three black pearls stand on the last row and three on the row behind. Count 9.

White 10. A second white pearl crosses. White leads by two points to none. Count 6.



	BEFORE		AFTER	
	●	○	●	○
+	0	2	2	2
x	2	0	2	0
Σ	9:3:3	6:2:2	5:2:3	6:2:2
∨	2	2	2	2

DIAGRAM 47. Black 11. c7–b8, e7–f8, b8–off, f8–off.

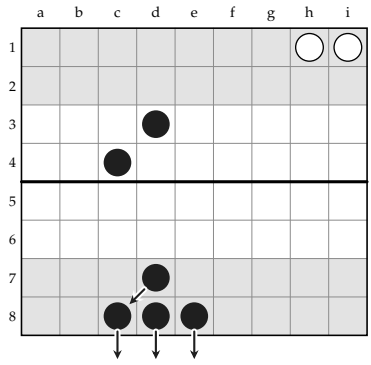


	BEFORE		AFTER	
	●	○	●	○
+	2	2	2	4
x	2	0	2	0
Σ	5:2:3	6:2:2	5:2:3	2:1:2
∨	2	2	2	1

DIAGRAM 48. White 11. g1–off, h1–off, g2–h1, h2–i1.

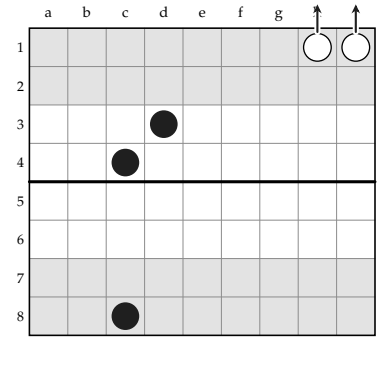
Black 11. Two pearls step across to the b- and f-files and cross. Black had three pearls on the last row that could have crossed instead, and it would have made no difference, since the count falls by four either way. Two points each. Count 5.

White 11. Two more cross, and the last two white pearls step down to the last row. White leads by four points to two and needs two steps.



	BEFORE ●	BEFORE ○	AFTER ●	AFTER ○
+	2	4	5	4
x	2	0	2	0
Σ	5:2:3	2:1:2	1:1:3	2:1:2
∨	2	1	1	1

DIAGRAM 49. Black 12. c8–off, d7–c8, d8–off, e8–off.

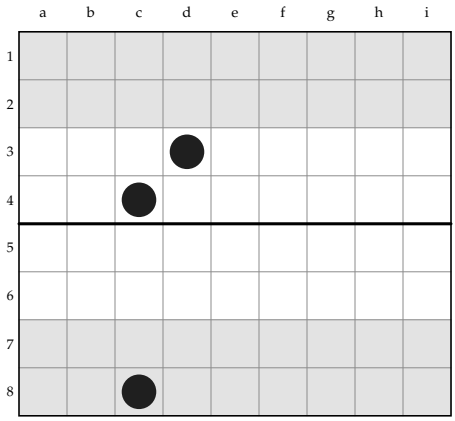


	BEFORE ●	BEFORE ○	AFTER ●	AFTER ○
+	5	4	5	6
x	2	0	2	0
Σ	1:1:3	2:1:2	1:1:3	0:0:0
∨	1	1	1	–

DIAGRAM 50. White 12. h1–off, i1–off.

Black 12. Black crosses three pearls and leads by five points to four. The other step brings a pearl to c8. It needs one more step to cross, and Black will not get it.

White 12. Both white pearls cross. White has no pearls left on the board, and the game ends there, with two of White's four steps unused.



	●	○
+	5	6
x	2	0
Σ	1:1:3	0:0:0
∨	1	–

DIAGRAM 51. The final position. White has scored six and has no pearls left, so the game is over. White wins by six points to five. The black pearl on c8 was one step from crossing.

29. What the Game Shows

Neither side lost a step. Every step either side took in this game was a forward step. Black made forty-eight of them and still lost. A player can do nothing wrong in the race and be beaten by what happens to the target.

It was decided on the seventh turn. Before Black's seventh turn Black was ahead on the count. After White's seventh turn White was ahead, and the remaining five turns only carried out the result. The cause was two black pearls standing beside a white pearl that White was glad to lose, and a capture that left White free to lose it.

The captures were not equal. Black gained one step on the count by capturing d5. White gained two steps on the count by losing d7. Both pearls count as captures by Black. The difference was in how far each had to go, and in which of Black's own pearls dropped out of the count in exchange.

The pearls left over scored nothing. Black ended with three pearls on the board and two captures. White ended with none of either. White won, because White had six points and Black had five.

One step was the margin. Black needed twenty-one steps after White's seventh turn and had time for twenty. In a game of this kind that is not

bad luck. It is the ordinary size of the difference between winning and losing, and it is the reason for counting.

Part Seven
Exercises

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30. Eight Questions

The answers are in the next section. Each question can be settled with the rules and methods of this article, and the reader is encouraged to settle it before looking.

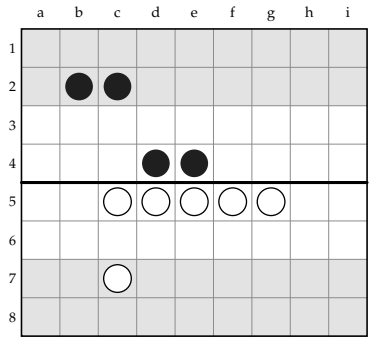


DIAGRAM 52. *Question 1.* White to move. Five white pearls face two black ones and neither is captured. Why not? Find the one step that captures both.

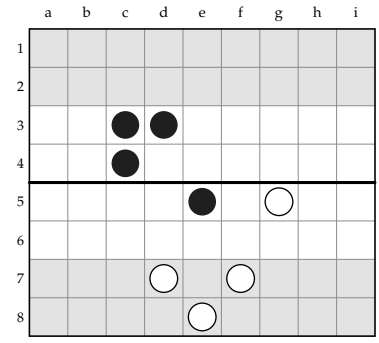


DIAGRAM 53. *Question 2.* Black to move. Which squares are shut to the pearl on e5? How can it still go forward?

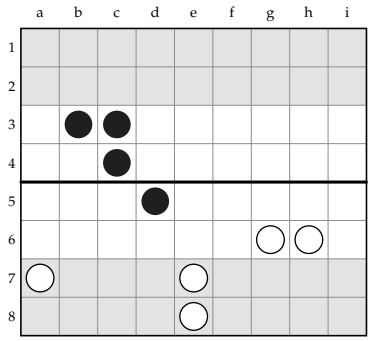


DIAGRAM 54. *Question 3.* Black to move. Can the pearl on d5 cross this turn?

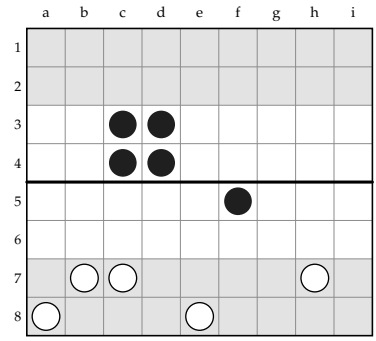
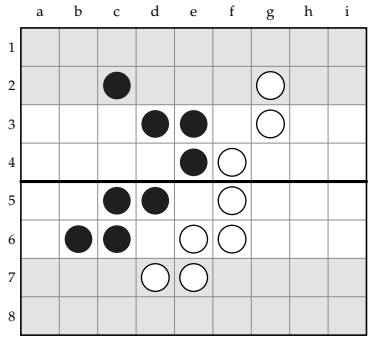
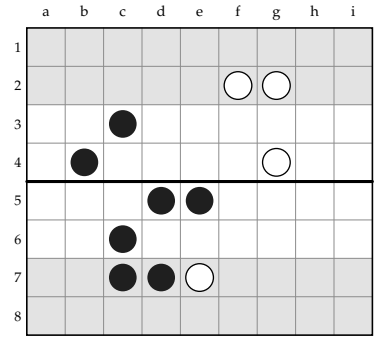


DIAGRAM 55. *Question 4.* White to move. Can White capture the pearl on f5?



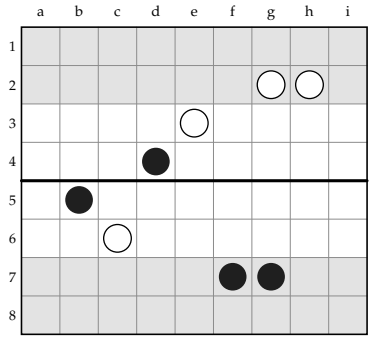
	●	○
+	0	0
x	0	0

DIAGRAM 56. *Question 5.* No pearl has been captured and none has crossed. Count both sides. Who is ahead if Black is to move? Who is ahead if White is?



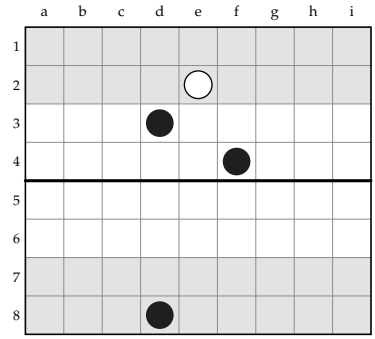
	●	○
+	1	1
x	3	0

DIAGRAM 57. *Question 6.* Each side has scored one point. What is the target? Who is ahead, with White to move?



	●	○
+	0	0
x	4	4

DIAGRAM 58. *Question 7.* Each side has made four captures and nobody has scored. Black to move. The step d4–d5 would capture the pearl on c6. Should Black play it?



	●	○
+	3	4
x	3	2

DIAGRAM 59. *Question 8.* Black has three points and three captures. White has four points and two captures. Black to move and win.

31. Answers

Question 1. Each black pearl has the other beside it and counts as two, so four white pearls are needed against either. Only three touch each. The step g5–f4 brings a fourth against e4, which is captured. The pearl on d4 is then alone with three white pearls touching it, and is captured by the same step.

Question 2. The squares e6 and f6 are shut. The first touches the white pearls on d7 and f7, and the second touches those on f7 and g5. The square d6 touches only d7, so the pearl can go forward by stepping there.

Question 3. Yes, but not in a straight line. The square d7 touches both white pearls and is shut. The pearl changes file: d5–c6, c6–c7, c7–c8 and off. A diagonal step forward loses no step, so the pearl loses nothing by going round.

Question 4. Yes. The pearl on h7 is one step from g6, and the pearl on e8 is two steps from e6. Both squares touch f5. That is three steps, and White has four: e8–e7, e7–e6, h7–g6 and one more. The black pearl was four files from the rest of its side and should not have been left there.

Question 5. Black's count is 38 and White's is 40. Each is ten turns. So whichever side is to move is ahead. A position cannot be judged without knowing whose turn it is.

Question 6. Black has one point and seven pearls, which makes eight. White has one point and four pearls, which makes five. The target is five. White needs four more crossings and has four pearls, so every one of them counts: 2, 2, 4 and 7, which is 15, or four turns. Black needs four more crossings and uses its nearest four pearls: 2, 2, 3 and 4, which is 11, or three turns. Black is ahead.

Nearly half of White's count is the single pearl on e7. Black must take care not to capture it. One black pearl touches it already. If a second arrived, the pearl would be taken, the target would fall to four, and the counts would be 8 for White and 7 for Black: two turns each, with White to move and ahead.

Question 7. No. As things stand the target is four and both counts are 13, which is four turns each. Black is to move and is ahead. Black should play four forward steps, and should keep the pearls from b5 and d4 from both touching c6 at the same moment, because a capture is not optional. It happens whenever the numbers come out.

If Black captures, the target falls to three. White no longer needs the pearl on c6, which was six of its thirteen steps, and needs only seven steps for the three pearls near home: two turns. Black can cross one pearl in the same turn as the capture, and then needs five steps more, which is also

two turns. White is to move. White finishes first and wins by three points to two. If Black runs instead, Black's fourth pearl crosses before White's does.

Question 8. The order of the steps decides it. If Black captures first, with f4–f3, White has no pearls left and the game ends at once, with White a point ahead. Black must first cross the pearl on d8, which levels the points at four each, and then play f4–f3. The game ends with the points level, and Black has made four captures to White's two.

Doing nothing does not help either. White's pearl is two steps from crossing, and when it crosses White will have five points and no pearls on the board.

Part Eight
Where to Go Next

32. Main Points

1. A turn is four steps and all four must be taken.
2. A pearl is captured when the enemy pearls touching it are at least twice its own side on the spot. Two enemy pearls capture a lone pearl, and four are needed against a pearl with one supporting pearl.
3. A pearl may not step into capture, but a step may cause another pearl of the same side to be captured.
4. The game ends when either side has no pearls on the board. Points decide it, and captures only settle level points.
5. Count the forward steps in every turn. Sideways loses one step and backward loses two.
6. Move in twos and threes. Check every lone pearl against the opponent's four steps.
7. One or two pearls block nothing. Five on alternate files stop every lone pearl, and stop no pair.

8. Take runners. Be careful about taking stragglers, and about standing next to them.
9. Both sides race to the target, which is the smaller of the two totals of points and pearls. Every capture lowers it.
10. Count before every capture, and every few turns in any case.

33. Further Reading

Four other articles in this volume take up subjects that are only introduced here.

Arithmetic Margrites develops the count of Part Four into a way of looking at the whole game. It works out exactly how a capture and a lost step change each side's count.

When to Capture goes through the capture in the opening, the middle of the game and the ending, from the point of view of the side ahead on the count and of the side behind.

The Level Lock deals with the standoff of Section 21: how the side that starts behind reaches it, how the other side avoids it, and what to do once it is on the board.

The Waiting Game examines a plan for the side that starts behind: keep every pearl back, let the opponent come, capture what can be captured, and score at the end.

The best next step, though, is to play. Set up arrangement C, count at the start, and count again every third turn.

Arithmetic Margrites

The game as a count of steps

Arithmetic Margrites is a way of looking at the game. Its claim is that one thing decides a game: which side will reach the score it needs in fewer turns. Everything else that players talk about, such as formations, walls, support, captures and space, matters only so far as it changes that sum, and can be measured by how much it changes it. Part One gives the method. Parts Two to Four go through the opening, the middlegame and the ending and restate the familiar ideas as changes to the count. Part Five gives four studies.

Part One
The Count

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1. The Claim

Most writing on Margrites treats the game as a contest of positions. One side has the better formation, or more space, or the safer pearls, and the better position is expected to win in the end. The arithmetic view starts from the other end. It asks what winning is, and works backward.

A game is won by having the higher score when it ends. It ends when one side has no pearls left on the board. A pearl scores by crossing, and crossing takes steps. Each side gets four steps a turn and has to play all four. So in a game the two sides play steps at the same rate, four at a time, and the winner is the side that reaches the score it needs first.

That is the whole of the view. A position is good if it needs fewer turns than the opponent's position. A move is good if it widens that difference

or keeps it. A formation is good if it stops your steps being wasted or makes the opponent waste some. Nothing else matters except through what it does to the turns.

This is a strong claim. Part of it is plain arithmetic: given the pearls, the scores and the side to move, and no interference, who wins is a matter of counting. The rest is a way of reading the fighting. Every block, capture and formation is treated as a way to change the count, and the steps it adds to or takes from each count can be read off the board. Parts Two to Four go through the game on that footing.

We call the side that will win the race, if both sides simply run, the side with the *edge*.

2. Distance, Count, Turns and Spare Steps

Four numbers are used throughout.

The *distance* of a pearl is the number of steps it needs to cross. A pearl on its last row has a distance of one. A pearl on its own front home row has a distance of seven, and a pearl on its back home row eight.

The *count* of a side is the sum of the distances of its pearls. Section 6 will narrow this to the pearls a side actually needs, but until then the count is all of them.

The number of *turns* a side needs is its count divided by four and rounded up. A count of 57, 58, 59 or 60 is fifteen turns. A count of 61 is sixteen.

The *spare* steps of a side are what is left over in its last turn: four times its turns, less its count. A count of 60 has no spare steps. A count of 57 has three.

The count is what you add up at the board. The turns are what matter. Two counts that round to the same number of turns are the same, and two counts one step apart can be a whole turn apart. Most of the practical skill in this method is knowing, at any moment, how far each side is from the next multiple of four. A player who knows that the opponent has no spare steps knows that a single step lost by the opponent adds a turn to the opponent's race. A player who has three spare steps, and knows it, can use three steps on something else without needing another turn.

Count in steps. Compare in turns.

3. The Side to Move

Black moves first, so Black's fifteenth turn comes before White's fifteenth. If the two sides need the same number of turns, Black finishes first. White finishes first only if it needs fewer turns than Black.

The same holds at any point in a game for whichever side is to move. Put in steps, it gives two rules to learn by heart.

1. The side to move wins the race if it is behind by no more than the opponent's spare steps.
2. The side not to move wins the race only if it is ahead by at least four steps, less the spare steps of the side to move.

Take the side to move with a count of 44 and the other side with 41. The side to move is three steps behind, and 41 has three spare steps, so both sides need eleven turns and the side to move finishes first. Now give the side to move 45 and the other side 44. The side to move is one step behind. Because 44 has no spare steps, the side to move needs twelve turns to the other side's eleven, and loses by a whole turn.

So the side to move does not gain a fixed number of steps by having the move. It can be behind by as many steps as the opponent has spare, which is anything from none to three. This is the first place where the arithmetic view says something a player would not guess. When you are to move, you can be as many steps behind as your opponent has spare steps.

4. The Arrangements

The count begins before the first step. White chooses how many pearls to put on its front row. Black may put on its front row at most two fewer than White does; it may put fewer, and when White puts two or fewer forward Black must put none. With Black taking the most the rule allows, there are nine arrangements, lettered by White's number.

	White	*Black*	*Count*		*Turns*		
	front/back	*front/back*	*W*	*B*	*W*	*B*	*Edge*
A	8/0	6/2	56	58	14	15	White
B	7/1	5/3	57	59	15	15	Black
C	6/2	4/4	58	60	15	15	Black
D	5/3	3/5	59	61	15	16	White
E	4/4	2/6	60	62	15	16	White
F	3/5	1/7	61	63	16	16	Black
G	2/6	0/8	62	64	16	16	Black
H	1/7	0/8	63	64	16	16	Black
I	0/8	0/8	64	64	16	16	Black

H and I need not be considered further. Black is all back in G, H and I alike, so in H and I White has put one or two more pearls on its back row and gained nothing for the steps they add to its count. On the arithmetic view White has no reason to choose them, and this article takes A to G as the arrangements of the game.

In each of A to G, White starts two steps ahead and Black has the move. By the first rule of Section 3, Black holds the edge when White has two or three spare steps, and White holds it when White has none or one. That is why the edge goes back and forth down the table. It has nothing to do with the shape of the arrangement. It follows from White's count and nothing else.

The table can be made more useful by asking how firmly the edge is held. For each arrangement, how many steps can the side with the edge lose before the edge changes hands, if the other side loses none?

	Edge	*In hand*	*Other side's spare*	*What the other side must do*
A	White	0	2	Make White lose one step and lose no more than two
B	Black	1	3	Make Black lose two and lose no more than three
C	Black	0	2	Make Black lose one and lose no more than two
D	White	1	3	Make White lose two and lose no more than three
E	White	0	2	Make White lose one and lose no more than two
F	Black	1	3	Make Black lose two and lose no more than three
G	Black	0	2	Make Black lose one and lose no more than two
H	Black	0	1	Make Black lose one and lose no more than one
I	Black	0	0	Make Black lose one and lose nothing

The edge is thin. In no arrangement does the side with the edge have more than one step in hand, and in six of the nine it has none. A game that starts with the edge can lose it to a single sideways step on the second turn. The side without the edge also always has at least as much room to waste steps as the side with it. In A to G the side behind has two or three spare steps. In H and I it has one or none, which is a further reason for White not to choose them. So the side behind can lose steps in making trouble without needing another turn, and the side ahead cannot lose steps in answering without losing the edge. The arithmetic view takes this to be the basic shape of every opening: one side has an edge that it loses with its first or second lost step, and the other has spare steps to use against it.

5. Lost Steps

A step that moves a pearl one row nearer its exit takes one off the count. Any other step is lost, in whole or in more than whole.

1. A sideways step leaves the count where it was. One step is lost.
2. A backward step puts one on the count. The step is lost and the row has to be made again, so two steps are lost.

Because all four steps must be played every turn, there is a simple and exact relation. If a side starts with a certain count and loses a certain

number of steps on the way, the turn on which it finishes is its starting count plus its lost steps, divided by four and rounded up. Nothing else enters into it, so long as no pearl is captured. Captures come in Section 6.

This turns a game without captures into a ledger with two columns. The table below is the ledger for arrangement D. Read down for the steps White has lost and across for the steps Black has lost. The letter shows who finishes first.

	Steps lost by Black								
White	0	1	2	3	4	5	6	7	8
0	W	W	W	W	W	W	W	W	W
1	W	W	W	W	W	W	W	W	W
2	**B**	**B**	**B**	**B**	W	W	W	W	W
3	**B**	**B**	**B**	**B**	W	W	W	W	W
4	**B**	**B**	**B**	**B**	W	W	W	W	W
5	**B**	**B**	**B**	**B**	W	W	W	W	W
6	**B**	**B**	**B**	**B**	**B**	**B**	**B**	**B**	W
7	**B**	**B**	**B**	**B**	**B**	**B**	**B**	**B**	W
8	**B**	**B**	**B**	**B**	**B**	**B**	**B**	**B**	W

The edge changes hands in bands. While White has lost no more than one step, White keeps the edge whatever Black has lost. If White has lost two to five, White needs Black to have lost at least four. If White has lost six or more, Black must have lost at least eight. And so on, four at a time. Every arrangement has a ledger like this, shifted by a step or two. A player of this school keeps the ledger for the game in progress in mind, and on each turn asks only where on it the two sides now stand.

A step diagonally forward is not a lost step. It takes a pearl one row nearer its exit, the same as a step straight forward. So a pearl can change file without losing a step while it is going forward. A sideways step loses a step when it has to be made on the spot, with no rows left in which to change file.

Three things cause lost steps, and the rest of this article is about them.

1. *Obstruction.* The square a pearl wants is occupied, or the pearl would be captured there, so it has to go round or wait.
2. *Compulsion.* You have to play four steps, and you have no forward step that is safe. So you play steps that do not count.
3. *Choice.* You go sideways or backward on purpose, to capture, to defend or to block.

6. The Target

So far the count has been the sum of all a side's pearls, as though both sides had to bring all eight home. That is true only while no pearl has been captured. Once pearls have been lost the sum changes, and the change is the most important thing in the method.

A side's *maximum* is its score plus its pearls on the board. It is the most it can finish with. Both sides start with a maximum of eight, and each pearl a side loses takes one off its maximum.

The *target* is the lower of the two maximums.

The reason is this. The game ends when one side has no pearls. The side with the lower maximum can never score more than its maximum, so the other side does not need more than that. If the other side reaches the same score, the scores are level and the captures decide, and the side with the higher maximum is the side that has lost fewer pearls and so made more captures. It wins the tie. So the side with the higher maximum needs only to reach the target. The side with the lower maximum needs the target as well, because the target is its own maximum, and it must get all its pearls home before the other side reaches the same score.

When the maximums are equal, the target is that number and both sides need every pearl. Then the game is a race to empty the board, as in the earlier sections.

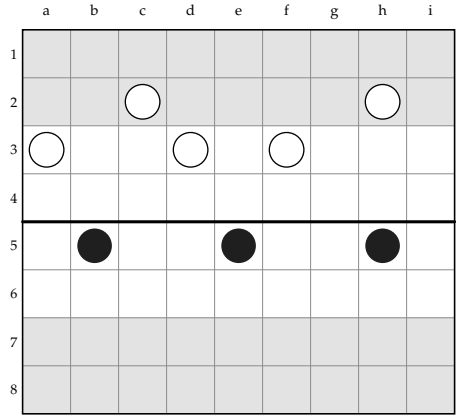
From here on, the count means the count to the target: the number of steps a side needs to raise its score to the target, using the pearls nearest to crossing. A pearl that is not needed for the target is *off the count*. The whole method is then one rule.

> *The target is the lower maximum. Each side counts its steps to the target. The side that needs fewer turns wins, and the side to move wins if the turns are equal.*

A game cannot be drawn under this rule, and it cannot be drawn in practice either. If the scores are level when the game ends, one side has pearls still on the board and the other has none, so the two have lost different numbers of pearls and the captures are unequal.

Wasting steps never helps in a pure race. A side that will lose by finishing can refuse to finish, but then the other side finishes instead and wins anyway.

Diagram 1 shows the rule at work. Black has scored two and has three pearls left, so Black's maximum is five. White has scored one and has five pearls, so White's maximum is six. The target is five. Black needs three



	●	○
+	2	1
x	2	3
Σ	12:3:0	10:3:2
∨	4	3

DIAGRAM 1. Black has scored 2 and White 1. Black has lost three pearls and White two.

more crossings and has three pearls, each four steps away: a count of twelve, which is three turns with nothing spare. White needs four more crossings and uses its four nearest pearls, at two, two, three and three: a count of ten, which is three turns with two spare. White's fifth pearl is off the count. It does not have to move again.

Both sides need three turns, so the side to move wins. If Black is to move, Black crosses its last pearl on its third turn, when White has had only two turns and has scored four. Black wins five to four. If White is to move, White reaches five on its third turn. Black then crosses its last pearl and the game ends five all, and White wins on captures, three to two.

Notice what White's ten steps did not have to include. White has five pearls and thirteen steps of distance on the board. A player who added them all up would think White was behind.

Part Two
The Opening

7. Choosing the Arrangement

The number of pearls White puts forward fixes the most that Black may put forward. So the arrangement is White's choice, and on the arithmetic view it is the most important choice White makes in the game, because it is the only time either side can set the count without playing a step.

The second table in Section 4 tells White what to choose. Six arrangements, B, C, F, G, H and I, give the edge to Black before a step is played. Two, A and E, give it to White with nothing in hand. One wasted step by White in either of them, with none by Black, and both sides need the same number of turns, and Black moves first. Only arrangement D gives White the edge and a step to lose.

The edge in D is small. In D White needs fifteen turns and Black sixteen. White has one spare step. Black has three. To take the edge Black has to make White lose two steps while losing no more than three itself. White, to keep it, must lose no more than one step in fifteen turns. If White loses two to five steps, it keeps the edge only if Black loses at least four.

A player who thinks in positions will say that this cannot be the basis of an opening. No one gets through fifteen turns against opposition and loses one step. That is true of both sides. Black will lose steps as well, and the ledger of Section 5 shows that what matters is the two totals and the band they fall in. The arithmetic player does not expect to keep the starting edge untouched. What such a player wants to know, on every turn, is which band the game is in and how far it is from the next one.

Black's choice of numbers is narrower. It can only hold pearls back, and each pearl held back adds a step to its count. In a pure race that never helps Black, so on the arithmetic view Black takes the most the rule allows. *The Level Lock*, in this volume, discusses the one reason Black might not. Black's opening is made of the shape of its eight pearls and of its first three or four turns. The rest of this Part is about what those turns can do to the count.

8. The Walk to the Center

The commonest plan in the opening is for both sides to bring their pearls up to the center line and close the front. We follow that plan through one turn at a time, counting as we go. At the end of it the edge has passed to the side that started behind, which most players do not expect.

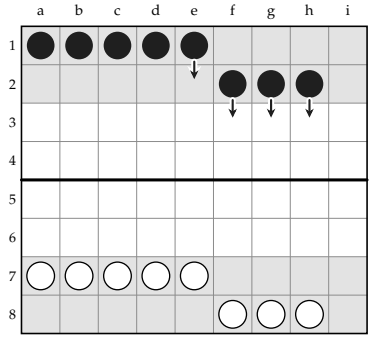
Take arrangement D. White's count is fifty-nine and Black's sixty-one. White needs fifteen turns and Black sixteen, so White has the edge.

Each side is going to stop at the center: White on its fifth row and Black on its fourth. Count the forward steps each side has before it gets there. We call these *free steps*.

1. White has five pearls on its front row, two steps from the line, and three on its back row, three steps from it. That is nineteen free steps.
2. Black has three pearls on its front row and five on its back row. That is twenty-one free steps.

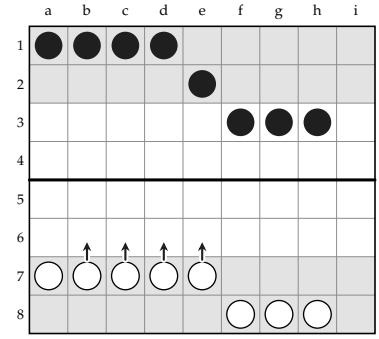
White has fewer free steps because White started nearer. Nineteen steps is four turns and three steps. Twenty-one is five turns and one step. So White's line will be complete first, and White will then have to go on playing four steps a turn with nowhere forward to play them.

The diagrams show the twelve turns. Each shows the position before the turn, with arrows for the four steps played.



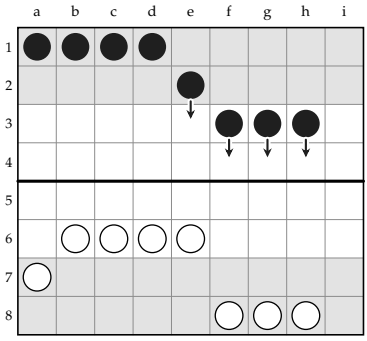
	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	61:16:3	59:15:1	57:15:3	59:15:1
∨	8	8	8	8

DIAGRAM 2. Black's first turn: f2–f3, g2–g3, h2–h3, e1–e2.



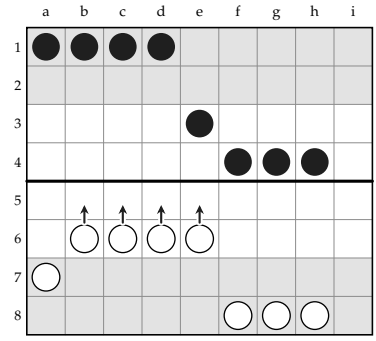
	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	57:15:3	59:15:1	57:15:3	55:14:1
∨	8	8	8	8

DIAGRAM 3. White's first turn: c7–c6, d7–d6, e7–e6, b7–b6.



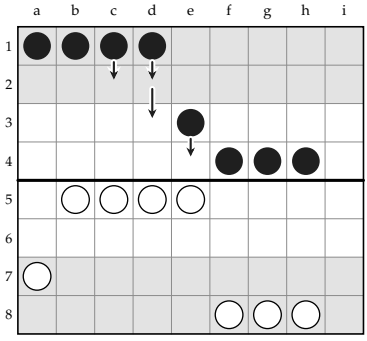
	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	57:15:3	55:14:1	53:14:3	55:14:1
∨	8	8	8	8

DIAGRAM 4. Black's second turn: f3–f4, g3–g4, h3–h4, e2–e3.



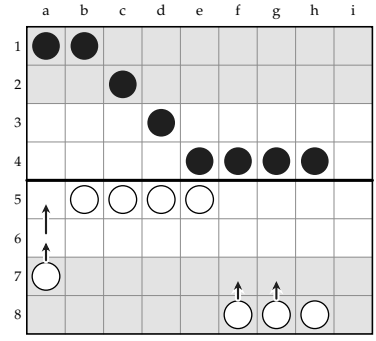
	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	53:14:3	55:14:1	53:14:3	51:13:1
∨	8	8	8	8

DIAGRAM 5. White's second turn: c6–c5, d6–d5, e6–e5, b6–b5.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	53:14:3	51:13:1	49:13:3	51:13:1
∨	8	8	8	8

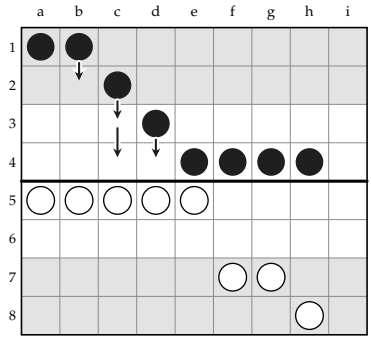
DIAGRAM 6. Black's third turn: e3–e4, d1–d2, d2–d3, c1–c2.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	49:13:3	51:13:1	49:13:3	47:12:1
∨	8	8	8	8

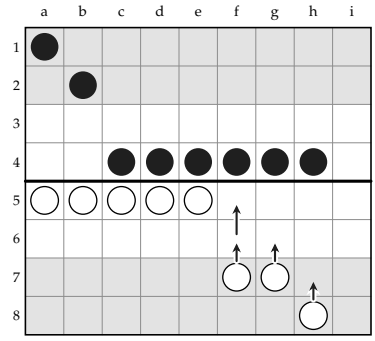
DIAGRAM 7. White's third turn: a7–a6, a6–a5, f8–f7, g8–g7.

For four turns every step either side plays is a forward step. Then the free steps begin to run out, and the two sides run out at different times.



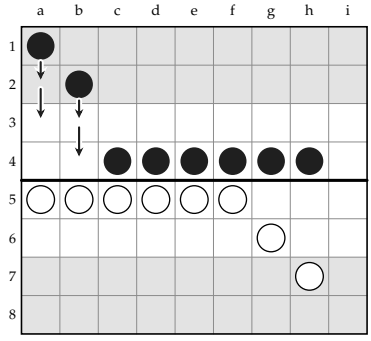
	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	49:13:3	47:12:1	45:12:3	47:12:1
∨	8	8	8	8

DIAGRAM 8. Black's fourth turn: d3–d4, c2–c3, c3–c4, b1–b2.



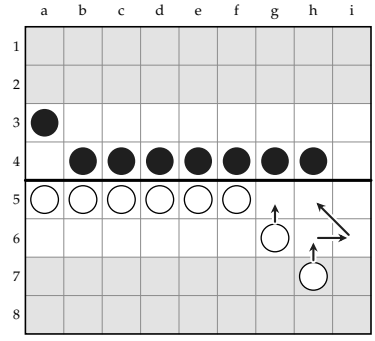
	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	45:12:3	47:12:1	45:12:3	43:11:1
∨	8	8	8	7

DIAGRAM 9. White's fourth turn: f7–f6, f6–f5, g7–g6, h8–h7.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	45:12:3	43:11:1	41:11:3	43:11:1
∨	8	7	6	7

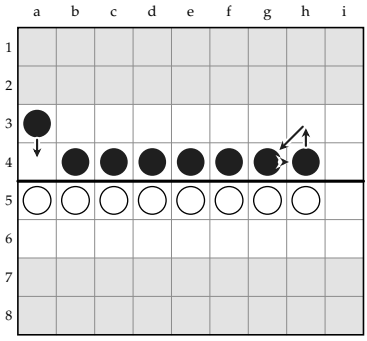
DIAGRAM 10. Black's fifth turn: b2–b3, b3–b4, a1–a2, a2–a3.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	41:11:3	43:11:1	41:11:3	40:10:0
∨	6	7	6	5

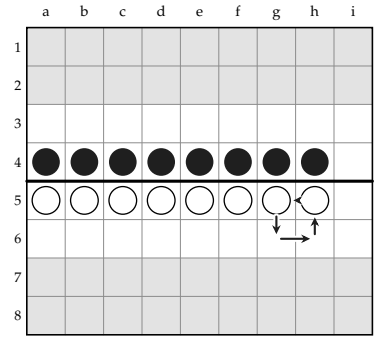
DIAGRAM 11. White's fifth turn: g6–g5, h7–h6, h6–i6, i6–h5.

	Black		*White*	
Turn	*forward*	*lost*	*forward*	*lost*
1 to 4	16	0	16	0
5	4	0	3	1
6	1	3	0	4
Total	21	3	19	5



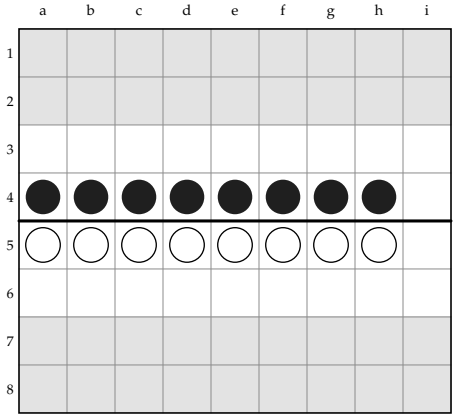
	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	41:11:3	40:10:0	40:10:0	40:10:0
∨	6	5	5	5

DIAGRAM 12. Black's sixth turn: a3–a4, h4–h3, g4–h4, h3–g4.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	40:10:0	40:10:0	40:10:0	40:10:0
∨	5	5	5	5

DIAGRAM 13. White's sixth turn: g5–g6, h5–g5, g6–h6, h6–h5.



	●	○
+	0	0
x	0	0
Σ	40:10:0	40:10:0
∨	5	5

DIAGRAM 14. After six turns each. The front is closed and level. Both counts are 40, and Black is to move.

Look at White's fifth turn. White has three forward steps left and must play four, so one pearl takes a sideways step on its way to the line. That is one step lost. On its sixth turn White's line is complete and it has no forward step at all. It plays four steps that change nothing: one pearl steps

back, the pearl beside it moves across, and the first comes forward again into the other's place. Four steps lost.

Black's fifth turn is all forward steps. On its sixth turn Black has one forward step left, plays it, and then has three steps to fill in the same way. Three steps lost.

So White has lost five steps and Black three. Put that into the ledger of Section 5. White started needing fifteen turns. Fifty-nine and five is sixty-four, which is sixteen turns. Black started needing sixteen. Sixty-one and three is sixty-four, sixteen turns. They now need the same number, and Black moves first. The edge has passed to Black, and neither side has done anything but walk forward.

9. The Level Lock

The position the walk ends in is the *level lock*: two full lines facing each other across the center. The same result can be reached by counting the position directly, without the history.

1. Every white pearl is on White's fifth row. It needs four steps to reach its last row and one to cross: five.
2. Every black pearl is on Black's fourth row. It also needs five.
3. Eight pearls at five steps each is forty. Both counts are forty.
4. Forty is ten turns. Both sides need ten turns.
5. Both sides have had the same number of turns, so Black is to move. With equal turns the side to move finishes first.

It does not matter which arrangement the game started from. Any arrangement that ends in a level lock ends at forty against forty with Black to move, because that is what the position is. In each of A to G White starts two steps ahead, and in every one of them White loses exactly two steps more than Black in walking to the center.

The count therefore makes the two sides level, and equal counts go to the side to move. That holds in a race, and a lock is not a race. While the front stays shut neither side can go forward. A side that marks time for a turn leaves the position as it was and hands the move to the other side, and with it the count's verdict. What decides a lock is which side has to open it, and the side that opens it loses steps in doing so.

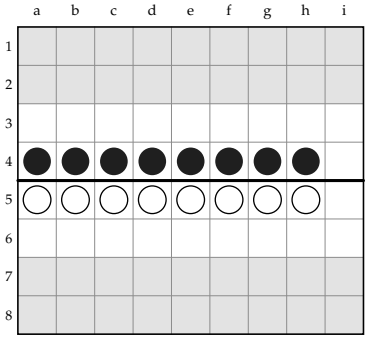
The side that has to open the lock is the side that runs out of free steps first. If both sides come straight up to the line, that is the side that started

the game ahead. In arrangement D, White started ahead, ran out first, and lost the edge on its sixth turn without doing anything but walk forward.

So on the arithmetic view the level lock is the natural aim of the side that starts behind, whichever color that is. *The Level Lock*, in this volume, discusses who has to open it in each arrangement, what the side ahead can do to avoid it, and what happens once it is opened.

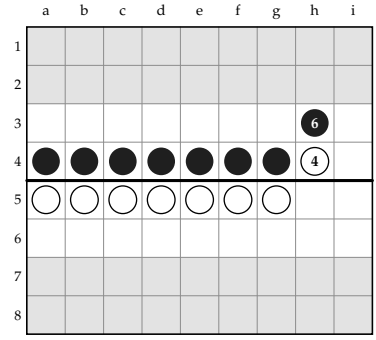
One caution goes with this. Forty is ten turns exactly, with no spare steps. Once the lock opens, a side that loses one step more than the other has lost a turn.

10. One Row Further Forward



	●	○
+	0	0
x	0	0
Σ	40:10:0	40:10:0
∨	5	5

DIAGRAM 15. The level lock. Every pearl is five steps from crossing. White 40, Black 40.



	●	○
+	0	0
x	0	0
Σ	41:11:3	39:10:1
∨	6	5

DIAGRAM 16. The same, but on the h-file White stands a row further forward. White 39, Black 41.

Now change one thing. In the second diagram the two pearls on the h-file have each moved one row toward Black's side. White's pearl is on its fourth row and Black's has been kept back on its third. Count again.

	White	*Black*
Seven pearls at five steps	35	35
The pearl on the h-file	4	6
Count	39	41
Turns	10	11

White needs ten turns and Black eleven. White has the edge, although Black is to move.

One row on one file changes the difference between the counts by two steps: one off the count of the side that gained the row and one on the count of the side that gave it up. Two steps is exactly what White started the game with. So a single row on a single file undoes the whole effect of the level lock.

The same sum applies to any number of rows. If one side's entire line stands a row further forward than a level lock, that is eight pearls at two steps each: sixteen steps, or four turns. Nothing else in the opening changes the counts by a quarter as much. When players say that a side "has more space", the arithmetic view says there is an exact figure for it: count the rows by which each pearl in the front is ahead of or behind the center, and double it.

> *A row gained on one file changes the difference between the counts by two steps. A row gained along the whole front changes it by four turns.*

A side does not gain a row by getting a single pearl there first. A pearl can stand only where it will not be captured, and a single pearl cannot stand next to two enemy pearls. A pearl that reaches the far side of the center by itself and finds two enemy pearls arriving beside it has to leave or be lost, and either way the steps it took are wasted or worse. A row is gained by arriving with enough beside you to stay.

A row gained also decides a lock. A level lock leaves the counts equal, and the lock then goes against the side that runs out of free steps first. A row gained on one file ends the equality outright: the side that gained it has the edge on the count, whoever is to move. For the side that would lose the lock, the opening is a matter of finding, among the nine files, the one where its pearls can get a row further than the opponent's can stop them.

Part Three

The Middlegame

11. The Steps a Runner Loses to Blockers

The middlegame begins when pearls get in each other's way. The arithmetic view asks how many steps a pearl loses to each obstruction, and the answer is usually fewer than players suppose.

A single pearl is refused a square in two cases: the square is occupied, or two or more enemy pearls stand next to it. We call such a square *shut*. Everything about blocking a single pearl comes down to which squares are shut and whether the pearl can avoid them while still going forward.

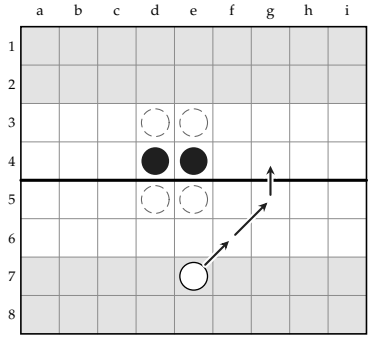


DIAGRAM 17. Two pearls side by side shut four squares. The runner passes without losing a step.

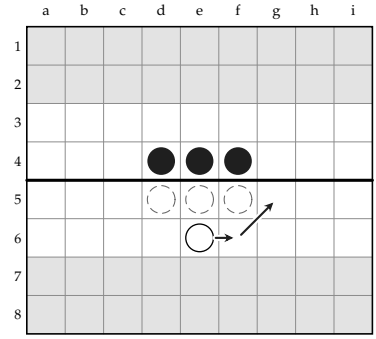


DIAGRAM 18. Three in a row shut all three squares in front of a pearl on the next row but one. It loses one step.

We worked out, for a number of standing formations, the fewest steps a single pearl must lose to get past and cross. The blockers stand on their fourth row and do not move. The pearl comes up the e-file and starts either three rows short of them or two.

Blockers	*Steps lost, starting*	
	three rows short	*two rows short*
One pearl, straight ahead	0	0
Two side by side	0	0
Two with a file between them	0	0
Three in a row	0	1
Five in a row, files c to g	1	2
Eight in a row, files a to h	2	3
Five on alternate files, a to i	*cannot pass*	

One or two pearls across the path block nothing. The pearl steps diagonally and goes by. Putting a pearl in front of an advancing pearl does not stop it. You lose the steps you use to place it, and the opponent loses none. There is one exception, two pearls standing one behind the other right in front of the runner, and Section 20 deals with it.

A single pearl loses a few steps to a solid line. A full line of eight pearls, with one file open at the end, makes a single pearl lose two or three steps going round, which is less than a turn. Against an opponent with no spare steps, those lost steps decide the race. A line of this kind does not hold a side up for the rest of the game, as the walls described in the textbooks are said to do.

It matters how early the obstacle is seen. For every formation that makes the pearl lose any steps, the figure in the right-hand column is one more than the figure beside it. A pearl that begins to change file a row sooner saves a step.

The last line of the table needs a section to itself.

12. Pairs

Five pearls on alternate files shut every square of three rows to a single pearl. Nothing gets past alone. It looks like the perfect wall, made with five pearls and leaving three to spare.

It does not survive two pearls arriving together. The numbers are these. A pearl with no supporting pearl is captured by two enemy pearls. Four enemy pearls are needed to capture a pearl with one supporting pearl, and six to capture a pearl with two. The same numbers apply to entering a square: a pearl with a supporting pearl beside it can step onto a square next to two or three enemy pearls and stay there.

Own pearls beside it	*Enemy pearls needed to capture it or keep it out*
None	2
One	4
Two	6
Three or more	cannot be done

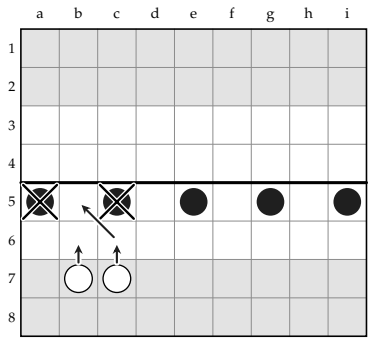


DIAGRAM 19. White to play. Three forward steps capture two pearls and open the line.

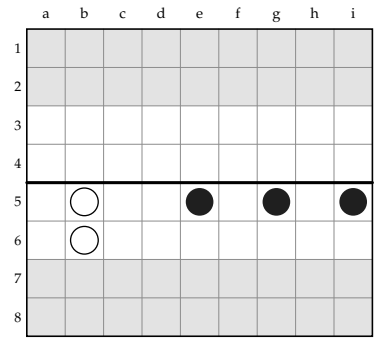


DIAGRAM 20. After the three steps. White has lost no steps and has one still to play.

In Diagram 19 two white pearls come up to the spaced line together. The first steps to b6, next to two black pearls, which it can do because its supporting pearl is beside it. The second steps to c6, and the black pearl on c5 now has two white pearls next to it and none of its own. It is captured. The second pearl goes on to b5 and the pearl on a5 is captured the same way. Three steps, all of them forward, and White still has one to play.

A side therefore has to choose. A pair takes apart a line that is spaced out to cover the board, and loses no steps in doing so. A line that is closed up so that a pair cannot take it apart covers only as many files as it has pearls, and the table in Section 11 shows that a runner loses one to three steps to it, once. Among the standing formations we worked out, none that holds against a pair makes a single pearl lose as much as a turn.

This is the arithmetic reading of support. A supported pearl has more free steps, because fewer squares are shut to it. An unsupported pearl runs out of free steps sooner and starts losing them.

Blockers do not stand still, of course. A line can move across to meet a pearl that is going round it. Every step a blocker takes to do that is a sideways step of its own, and it is lost. A side that shifts three pearls one file to make a runner lose one more step loses three steps to make the opponent lose one. Sometimes that is right. Section 21 discusses when.

13. The Capture: What It Changes

Whether to capture is usually decided by a set of questions about lost steps, score and the end of the game. The arithmetic view reduces these to one sum. To see where the sum comes from, take a capture apart and follow what it does to each number in turn.

Suppose you capture a pearl, and suppose the captures were level before, or you were already ahead. Five things happen.

One: the opponent's maximum falls by one. A maximum is the score plus the pearls on the board. The opponent has one pearl fewer.

Two: the target falls by one. The target is the lower of the two maximums. The opponent's maximum was the lower, or equal to yours, and it has just fallen. So the number both sides are racing to is one less than it was.

Three: the opponent's count falls. The opponent needed every pearl it had. One of them is gone, and so are the steps that pearl still had to make. The opponent's count falls by the distance of the pearl you took. We call that distance the *near figure*, because the pearls that get captured are usually near ones.

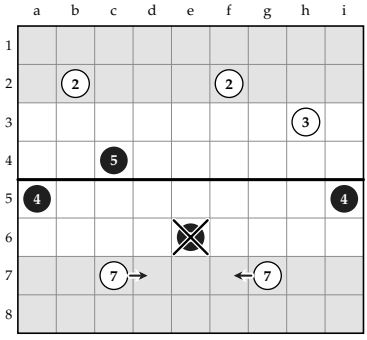
Four: your count falls. You need one crossing fewer than before. So one of your pearls is no longer needed, and the one to drop is the slowest. It comes off the count. Your count falls by its distance. We call that the *far figure*.

Five: you may have lost steps. Any step of the capture that did not go forward is a lost step, and counts against you. Notice that both counts fall. A capture lowers the counts of both sides. That is the fact players find hardest to accept, and it is the reason a side can lose by capturing. What matters is which count falls further.

> *The steps a side gains by a capture are the far figure, less the near figure, less the steps it loses in making the capture.*

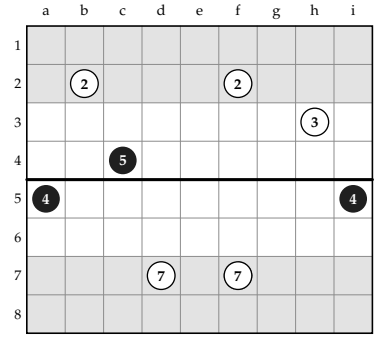
The far figure is yours and you want it large. The near figure is the opponent's and you want it small. So you gain most by capturing a pearl that is close to crossing, at a time when your own slowest pearl is far from it.

14. A Capture Worked Through



	●	○
+	2	1
x	2	2
Σ	16:4:0	21:6:3
∨	5	7

DIAGRAM 21. Before. Black has scored 2 and White 1. Each side has lost two pearls. The number on a pearl is its distance. White to play.



	●	○
+	2	1
x	2	3
Σ	13:4:3	14:4:2
∨	5	7

DIAGRAM 22. After the capture. Black has lost three pearls. One white home pearl, ringed, is off the count.

In the first diagram White is to play and looks badly behind. Two white pearls have never left the home row. A black pearl has come down between them and stopped on e6. White can capture it with two sideways steps, c7–d7 and g7–f7.

Count the position before the capture.

Before	*Black*	*White*
Score	2	1
Pearls on the board	4	5
Maximum	6	6
Target	6	
Crossings still needed	4	5
Pearls counted	3, 4, 4, 5	2, 2, 3, 7, 7
Count	16	21
Turns	4	6

White is two turns behind. Fourteen of its twenty-one steps belong to the two home pearls.

Now make the capture and go through the five changes.

1. Black's maximum falls from six to five.

2. The target falls from six to five.
3. Black needed all four pearls and now has three. The pearl on e6 was three steps from crossing. Black's count falls by three, from sixteen to thirteen. The near figure is three.
4. White needed five crossings and now needs four. It has five pearls, so one is not needed. The slowest is a home pearl, seven steps from crossing. White's count falls by seven, from twenty-one to fourteen. The far figure is seven.
5. Both capturing steps were sideways. White has lost two steps.

After	*Black*	*White*
Score	2	1
Pearls on the board	3	5
Maximum	5	6
Target	5	
Crossings still needed	3	4
Pearls counted	4, 4, 5	2, 2, 3, 7
Count	13	14

The sum: seven, less three, less two. White gains two steps by the capture.

Two steps does not sound like enough to recover two turns. The reason it is enough lies in the turns, not the steps. White has played two steps of its turn and has two left. It uses them to cross the pearl on b2. That leaves White needing twelve steps, which is three turns. Black needs thirteen, which is four, and it is Black's move. White now finishes a turn sooner.

	Black needs	*White needs*
Before the capture, White to move	4 turns	6 turns
After White's turn, Black to move	4 turns	3 turns

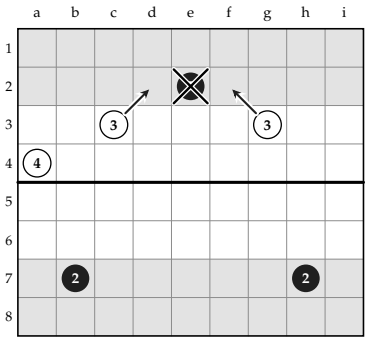
Black's three steps saved did not change its turns: sixteen and thirteen are both four. White's seven changed everything. A player looking at the first diagram without the count sees a side two turns behind. A player with the count sees a side with a far figure of seven, and looks for something to capture.

15. Four Captures Compared

The same sum gives very different answers in different positions. Four cases cover most of what happens.

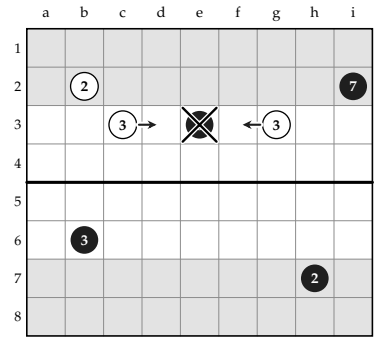
The runner. This is the capture just worked through. The pearl taken is close to crossing, so the near figure is small. If your slowest pearl is still at home the far figure is seven or eight. Taking a runner three steps from home gains four or five steps, before any steps lost in making the capture.

The straggler. A pearl that has not left its home rows has a distance of seven or eight. Your own slowest pearl can be no further than eight. So taking it gains one step at the very most, and usually the side that loses the pearl gains more.



	●	○
+	3	3
x	2	2
Σ	11:3:1	10:3:2
∨	7	4

DIAGRAM 23. The straggler. Near figure 7, far figure 4, no steps lost. Black gains three steps by the capture.



	●	○
+	3	2
x	3	1
Σ	5:2:3	8:2:0
∨	3	3

DIAGRAM 24. The spare pearl. Black has scored 3 and lost one pearl. White has scored 2 and lost three. Black was not counting the pearl on e3.

In the first diagram the score is three all and each side has lost two pearls. White can take the black pearl on e2 with two forward steps. White loses no steps in making the capture. The pearl is seven steps from crossing, and White's slowest pearl is four. Black's count falls from eleven to four. White's falls from ten to six. Before, both sides needed three turns and White was to move. After, Black needs one. White has taken off the board the one pearl that was keeping Black from finishing. Study 1 plays it out.

Several captures in one turn. When several pearls are taken in one turn, each takes another of your slow pearls off the count. If you take three pearls that stand four steps from crossing, and your three slowest are at

eight, eight and seven, the far figures come to twenty-three and the near figures to twelve. The side that makes the three captures gains eleven steps, before any steps lost in making them. Taking several pearls of an advanced line is right for the same reason that taking a runner is right. Taking three pearls on their home rows would gain nothing. Study 3 is a turn of this kind.

The spare pearl. All of the above assumed the captures were level or in your favor. If the opponent is ahead in captures, things are different. The opponent's maximum is the higher one, so the target is your own maximum and a capture does not move it. Your count does not change at all. The opponent has more pearls than it needs, and is counting only the nearest.

In the second diagram Black has lost one pearl and White three. Black's maximum is seven and White's five, so the target is five. Black has scored three and needs two more crossings. It is counting the pearls at two and three. The pearls at six and seven are spare.

	Black	*White*
Pearls counted before	2, 3	2, 3, 3
Count before	5	8
Pearls counted after the capture	2, 3	2, 3, 3
Count after, with White's two lost steps	5	10

White takes the pearl on e3 with two sideways steps. Nothing in Black's sum changes, because Black was not using that pearl. Nothing in White's changes either, except that White has lost two steps. Eight was two turns. Ten is three. White was winning the race and now loses it.

> *When the opponent has pearls to spare, taking one of them gains nothing. Taking one it was counting gains only the difference between that pearl and the spare one that replaces it.*

A side that is behind in captures should therefore look only at the opponent's nearest pearls. Those are the ones on the count.

One more thing follows from the sum. The far figure belongs to the side making the capture. Two sides can make the same capture and gain different numbers of steps from it. A side with two pearls still at home gains more from any capture than a side whose pearls are all past the center. That is one reason not to hurry the last pearls up.

16. The Steps Lost in Making a Capture

The third term in the sum is the steps lost in making the capture. They are counted exactly as in Section 5, with one addition.

1. A capturing step that goes forward loses nothing.
2. A capturing step that goes sideways loses one step.
3. A capturing step that goes backward loses two steps, if the pearl is on the count.
4. A step by a pearl that is off the count loses one step, whichever way it goes.

The fourth line is new. A pearl off the count does not have to get home, so it does not have to make up the ground it gives. Its step is simply one of the turn's four that did not take anything off the count. For a side with pearls off the count, those pearls are the ones to send back after a runner. A backward step by such a pearl loses one step, not two.

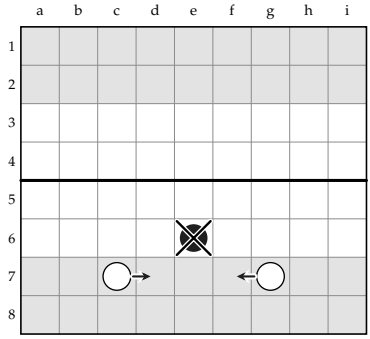


DIAGRAM 25. Two sideways steps. Two steps lost.

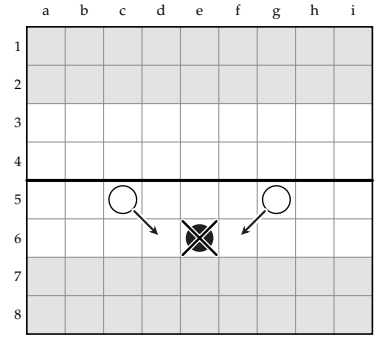


DIAGRAM 26. Two backward steps. Four steps lost, if both pearls are on the count.

The two diagrams show the same capture made with different numbers of lost steps. In Diagram 25 the capturing pearls are still on their home row and step sideways. In Diagram 26 they have gone forward and have to come back. Players learn that a capture finished on the first step of a turn is usually right, that one taking all four steps seldom is, and that captures needing a backward step are to be distrusted. The arithmetic view says these are one piece of advice. Steps lost in making a capture are subtracted from what the side gains by it, and backward steps are subtracted twice.

17. Counting a Capture at the Board

The whole of the last four sections comes down to a short routine, to be gone through before any capture.

1. *Who is ahead in captures?* If the opponent is, ask whether the pearl you can take is one it is counting. If it is not, stop. You gain nothing by taking it.
2. *Find the near figure.* How many steps is the pearl from crossing?
3. *Find the far figure.* How many steps is your own slowest counted pearl from crossing?
4. *Find the lost steps.* One for each sideways step, two for each backward step by a counted pearl, one for any step by a pearl off the count.
5. *Do the sum.* Far figure, less near figure, less lost steps.
6. *Turn it into turns.* Work out each side's count after the capture and divide by four. A gain in steps that leaves the turns as they were has not yet changed the race, and neither has a loss in steps that leaves them as they were.

The last step is the one most often left out. In the capture worked through above, the sum came to two steps and the result was three turns.

18. Giving Up a Pearl

A pearl can be given up on purpose. Seen from the count, giving up a pearl is the capture of Section 13 looked at from the other side, with the difference that the side losing the pearl chooses it.

Suppose your maximum is the lower one, or the two are equal. You let a pearl be captured. The target falls by one. Your count falls by the distance of the pearl you gave up. The opponent's count falls by the distance of the opponent's slowest counted pearl.

> *Give up a pearl when it is further from crossing than the opponent's slowest counted pearl, and the difference is enough to change the turns.*

The pearl to give up is your slowest, if it can be done. A pearl seven steps from home, given up against an opponent whose slowest counted

pearl is three steps from home, gains you four steps. That is a turn. Study 2 shows it deciding a game.

If your maximum is the higher one, giving up a pearl gains nothing. If the pearl was off the count, nothing changes. If it was on the count, you have to bring up a slower one in its place. The side that is ahead in captures has no pearls it is better off without. A pearl it does not need is off the count, and losing that pearl changes nothing.

Two limits apply, and both come from the rules. A pearl can be given up only where enemy pearls stand next to it, since it is captured by removing its support. And a side cannot give up its last pearl.

Part Four
The Ending

19. Who Wants the Game to End

The target rule sorts the two sides into different jobs.

The side with the lower maximum has to end the game. Its only way to win is to get every one of its pearls off the board before the other side reaches the target. It gains nothing by waiting, because the other side reaches the target in the meantime, and after that there is nothing the first side can do. Every pearl it has left on the board is a pearl it must cross or get rid of.

The side with the higher maximum does not have to end the game. It has to reach the target. Once it has, it has won, and what happens to its remaining pearls does not matter. They can cross, or stand, or be captured. If they are all captured, the game ends with that side at the target and the other below it. If the other side then crosses everything, the scores are level and the side that reached the target first has the captures.

When the maximums are equal, both sides have the first job. Each needs every pearl home and the first to manage it wins.

Before any capture, ask which side wants the game to end. The arithmetic view gives the answer in advance. The side with the lower maximum always does, and a side with equal maximum does if it is ahead on the count. Any step that brings the end nearer is good for that side, whoever

plays it. This is why a capture can lose. It can bring the end nearer for a side that wanted the end.

20. The Edge of the Board and the Last Rows

Section 11 found that a pearl loses few steps to standing blockers, because a pearl can go round them on the diagonal while it still has rows to go forward in. In the ending this turns round. A pearl two or three steps from crossing has no rows left to go round in. On the last rows a runner that has to go round a block loses every sideways step it takes.

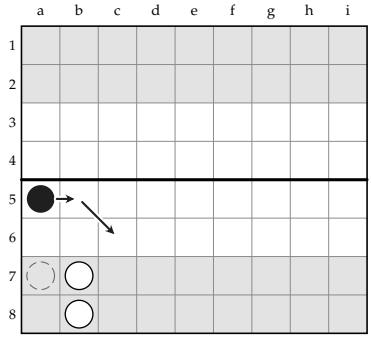


DIAGRAM 27. On the edge, two pearls are enough. The runner loses a step.

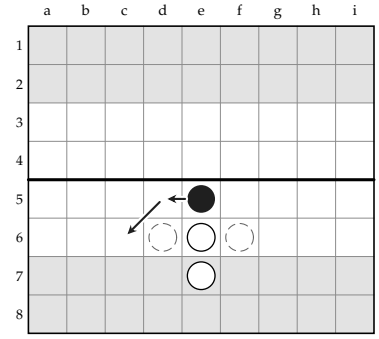


DIAGRAM 28. In the middle, two pearls do it only like this, one behind the other. The runner loses a step.

In Diagram 27 a black runner on the edge file is four steps from crossing and would cross this turn. Two white pearls on their home rows shut the square it must pass through. It has to step sideways first, and that is five steps, which is two turns. In Diagram 28 the runner is in the middle of the board. Two pearls can make it lose a step there in only one way: standing one behind the other, directly in front of it. We tried every other placing of two pearls and the runner slips past all of them on a diagonal. Three in a row across its path will also do it. On the edge file it is another matter. There are sixteen ways to place two pearls that make a runner on a5 lose at least one step, and six of them make it lose two or three.

In both diagrams the runner loses nothing if it starts one row further back. From there it sees the block in time and changes file on the way. So the block has to be made late, when the runner is already on the row in front. A block made early is a block the runner walks round.

Three rules for blocking in the ending follow.

1. Block the square, not the file. What matters is that every forward square of the runner is shut on the turn it wants to step.
2. Block near your own home rows, where the runner has no room.
3. Block on the edge when you can. Two pearls can do it there in many ways, and in the middle in one.

For the side with the runner the rules are the reverse. Keep runners off the edge files if the opponent has a pair at home. Change file early. And if the forward squares are going to be shut next turn, find out now, while a diagonal step still loses nothing.

21. Using Spare Steps

The side that makes a block loses steps. Its pearls have to get to their squares, and those steps are usually sideways. Whether a side should make the block depends on two numbers: your own spare steps and the opponent's.

You lose nothing by using your own spare steps. If you need ten steps to reach the target, you need three turns, and you will play twelve steps in them. Two will be wasted whatever you do. You can use those two steps for anything without needing another turn. Steps lost beyond your spare steps add a turn for every four, or part of four.

The opponent needs another turn only once it has lost more steps than it has spare. If the opponent has no spare steps, one lost step adds a turn. If the opponent has two spare, making it lose one step does not change its turns, though it leaves the opponent one spare step instead of two.

	Opponent's spare steps			
Steps you make the opponent lose	0	1	2	3
1	1 turn	none	none	none
2	1 turn	1 turn	none	none
3	1 turn	1 turn	1 turn	none
4	1 turn	1 turn	1 turn	1 turn
5	2 turns	1 turn	1 turn	1 turn

The best case is plain from the table. You have spare steps and the opponent has none. Then anything that makes the opponent lose a single step, while you lose no more than your spare steps, gains you a turn without adding one of your own. Study 4 is a case of this.

The same reasoning tells you when to leave a block alone. If you have no spare steps, making a block adds a turn to your race. You gain by it only if it makes the opponent lose more than a turn, and from the table that means at least five steps more than the opponent has spare. That almost never happens.

> *Know your spare steps and the opponent's before every turn of the ending. You can use yours without needing another turn. The opponent needs another turn only once it has lost more steps than it has spare.*

Part Five
Studies

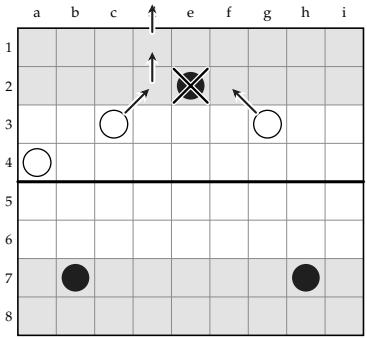
Each study gives a position and the count, and then the play. A diagram shows the position before the turn, with arrows for the four steps played in it. The last diagram of a study shows where the play ends. The positions are made up to show one point each.

Study 1
A Capture That Loses the Game

The score is three all, each side has lost two pearls, and White is to play. The first diagram below shows the position. Black has two pearls two steps from crossing and one that has never left its home rows. White has pearls at three, three and four. The maximums are equal and the target is six.

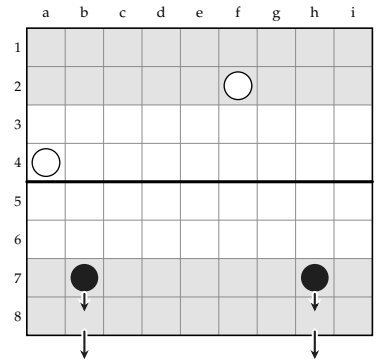
On the count White wins by running. White needs ten steps and Black eleven. Both need three turns and White moves first.

The black pearl on e2 is alone, and two white pearls can reach it going forward. Both steps take one off White's count, so White loses no steps in making the capture.



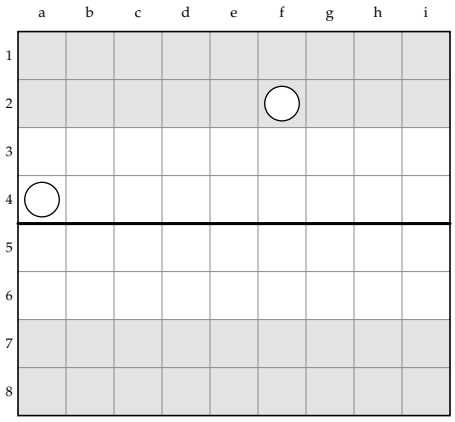
	BEFORE		AFTER	
	●	○	●	○
+	3	3	3	4
x	2	2	2	3
Σ	11:3:1	10:3:2	4:1:0	2:1:2
∨	7	4	2	2

DIAGRAM 29. White's first turn: c3–d2, g3–f2, d2–d1, d1–off. The pearl on e2 is captured.



	BEFORE		AFTER	
	●	○	●	○
+	3	4	5	4
x	2	3	2	3
Σ	4:1:0	2:1:2	0:0:0	2:1:2
∨	2	2	–	2

DIAGRAM 30. Black's first turn: b7–b8, b8–off, h7–h8, h8–off.



	●	○
+	5	4
x	2	3
Σ	0:0:0	2:1:2
∨	–	2

DIAGRAM 31. The end. Black has no pearls left and wins five to four.

White loses the game by it. Work the sum of Section 13. The pearl taken was seven steps from crossing: the near figure is seven. White's slowest pearl was four steps from crossing: the far figure is four. The sum is four less seven, so Black gains three steps by the capture.

In turns it is worse than that. After the capture the target is five. Black needs two crossings and has two pearls two steps away: four steps, one turn. White also needs two crossings, but has only two steps of its turn left and crosses one pearl. Black then crosses both of its pearls and the game is over.

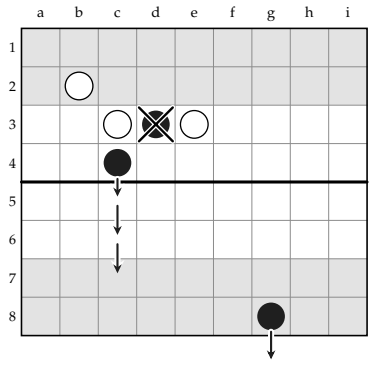
White took off the board the one pearl that was keeping Black from finishing. The count shows it in advance: the near figure was larger than the far figure.

Study 2

Giving Up a Slow Pearl

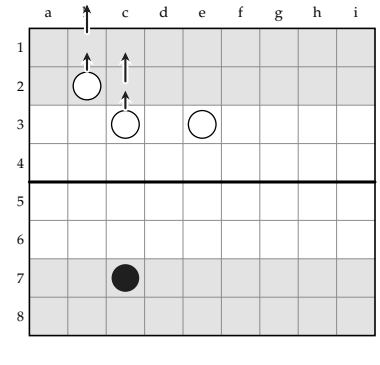
The score is three all, each side has lost two pearls, and Black is to play. The target is six and both sides need all three pearls. White's count is eight, two turns. Black's is twelve, three turns. Black is to move and is a turn behind.

The black pearl on d3 is six steps from crossing. It has white pearls on both sides and is kept on the board by the black pearl on c4. If that pearl steps away, the pearl on d3 is captured.



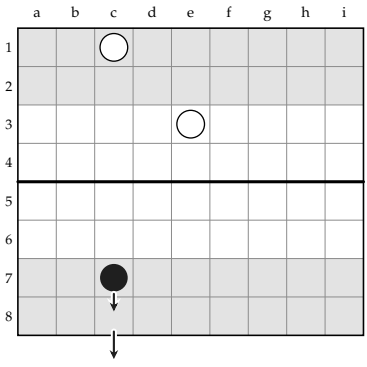
	BEFORE		AFTER	
	●	○	●	○
+	3	3	4	3
x	2	2	2	3
Σ	12:3:0	8:2:0	2:1:2	5:2:3
∨	6	3	2	3

DIAGRAM 32. Black's first turn: c4–c5, g8–off, c5–c6, c6–c7. The pearl on d3 is captured.



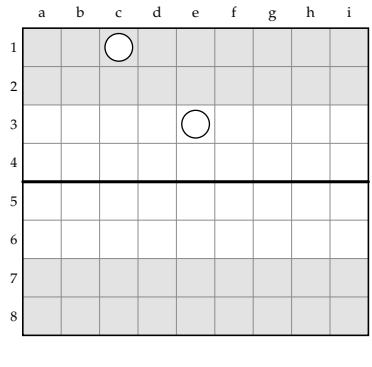
	BEFORE		AFTER	
	●	○	●	○
+	4	3	4	4
x	2	3	2	3
Σ	2:1:2	5:2:3	2:1:2	1:1:3
∨	2	3	2	1

DIAGRAM 33. White's first turn: b2–b1, b1–off, c3–c2, c2–c1.



	BEFORE		AFTER	
	●	○	●	○
+	4	4	5	4
x	2	3	2	3
Σ	2:1:2	1:1:3	0:0:0	1:1:3
∨	2	1	–	1

DIAGRAM 34. Black's second turn: c7–c8, c8–off.



	●	○
+	5	4
x	2	3
Σ	0:0:0	1:1:3
∨	–	1

DIAGRAM 35. The end. Black has no pearls left and wins five to four.

Black's first step gives the pearl up. The target is now five. Black crosses one pearl and brings the other within two steps of home. White needs two more crossings, from pearls at two and three, which is five steps. It has four. Black crosses its last pearl first.

The sum is that of Section 18. Black gave up a pearl six steps from crossing. White's slowest counted pearl was three steps from crossing, and came off White's count. Six less three is three steps to Black. Before, Black needed three turns to White's two. After, each needed two, and Black was to move.

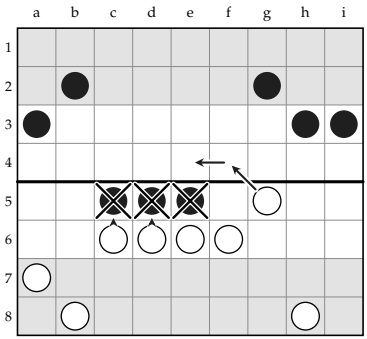
The step that gave the pearl up was a forward step by a pearl that had to move anyway, so Black lost no step by it. And the chance depended on the white pearls standing next to d3. A turn later they would have gone.

STUDY 3

Three Captures in One Turn

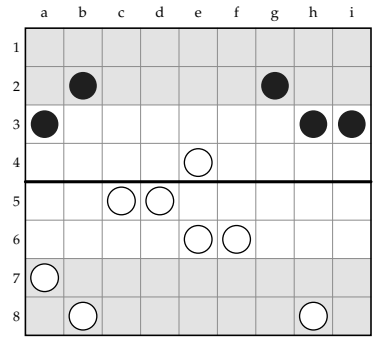
Nothing has been scored and all sixteen pearls are on the board. White is to play. Three black pearls have come down to the center and four white pearls face them. White has three pearls at home.

White's count is fifty-two, thirteen turns. Black's is forty-four, eleven turns. White is two turns behind and it is White's move.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	3
Σ	44:11:0	52:13:0	32:8:0	26:7:2
∨	7	8	7	6

DIAGRAM 36. White's first turn: g5–f4, f4–e4, d6–d5, c6–c5. The pearls on e5, d5 and c5 are captured.



	●	○
+	0	0
x	0	3
Σ	32:8:0	26:7:2
∨	7	6

DIAGRAM 37. After the turn. Black needs 32 steps, eight turns. White needs 26, seven turns.

The pearl on e5 had one supporting pearl and three white pearls next to it. The first step makes four and it is captured. The pearl on d5 then has one supporting pearl and three white pearls next to it. The second step makes four. It is captured, and the pearl on c5, left alone beside two white pearls, goes with it. White plays its last two steps forward.

Now the sum. The near figures are four, four and four: twelve. White's three slowest pearls were at eight, eight and seven, and all three come off the count: the far figures are twenty-three. One step was sideways. Twenty-three, less twelve, less one, is ten steps to White.

Check it on the board. The target is five. Black has five pearls left, at six, six, six, seven and seven: thirty-two steps, eight turns. White needs five crossings from its five nearest pearls, at four, five, five, six and six: twenty-six steps, seven turns. White was two turns behind and is one ahead.

White gained so many steps because the pearls taken were advanced, so the near figures were small, and because White had three pearls at home, so the far figures were large. White's backward pearls, which looked like a weakness, gave the capture its large far figures.

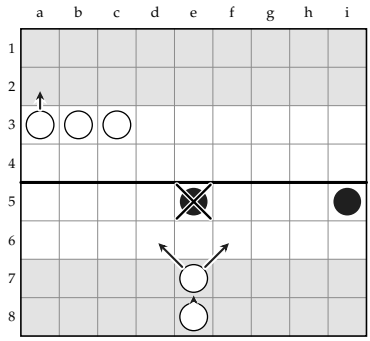
STUDY 4

Blocking a Runner Instead of Capturing It

Black has scored three and lost three pearls. White has scored two and lost one. White is to play. Black's maximum is five and White's is seven, so the target is five. Black needs both pearls home: eight steps, two turns, nothing spare. White needs three crossings from its three pearls on the third row: nine steps, three turns, three spare. White's two pearls on the e-file are off the count. White is to move and is a turn behind.

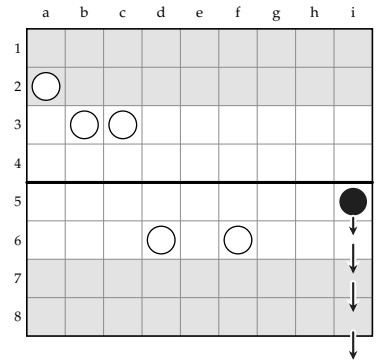
White has three spare steps, and two ways to use them.

The capture. The black pearl on e5 is alone. Three steps by the two pearls off the count put a white pearl on each side of it. They come out of White's spare steps, so White needs no extra turn for the capture.



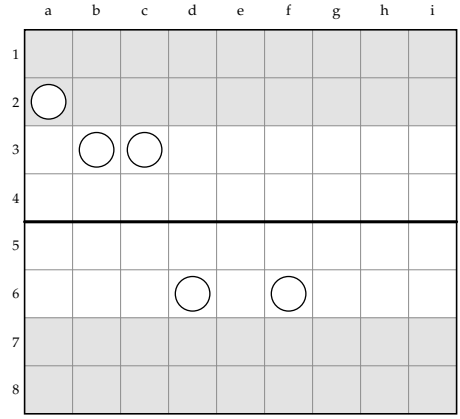
	BEFORE		AFTER	
	●	○	●	○
+	3	2	3	2
x	1	3	1	4
Σ	8:2:0	9:3:3	4:1:0	5:2:3
∨	4	3	4	3

DIAGRAM 38. White's first turn: e7–d6, e8–e7, e7–f6, a3–a2. The pearl on e5 is captured.



	BEFORE		AFTER	
	●	○	●	○
+	3	2	4	2
x	1	4	1	4
Σ	4:1:0	5:2:3	0:0:0	5:2:3
∨	4	3	–	3

DIAGRAM 39. Black's first turn: i5–i6, i6–i7, i7–i8, i8–off.

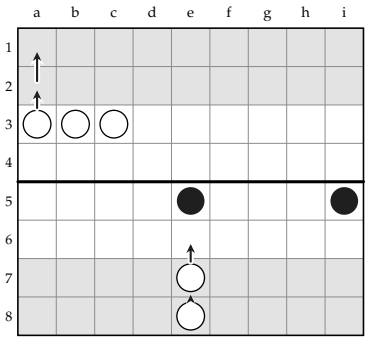


	●	○
+	4	2
x	1	4
Σ	0:0:0	5:2:3
∨	–	3

DIAGRAM 40. The end. Black has no pearls left and wins four to two.

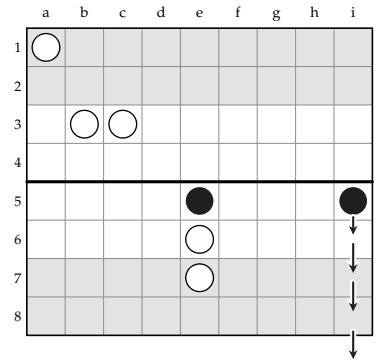
White loses at once. Black's maximum falls to four, and the target with it. Black now needs only the pearl on i5, four steps from crossing. By the capture White removed the one pearl that stood between Black and the end of the game. The side with the lower maximum wants the game to end, and any step that brings the end nearer is good for that side, whoever plays it.

The block. Two steps put the same two pearls one behind the other, directly in front of the runner. This is Diagram 28. Both squares diagonally in front of the black pearl are shut, and it has to step sideways before it can go on.



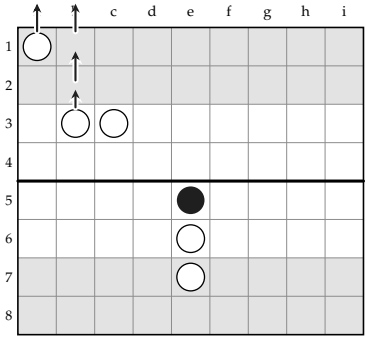
	BEFORE		AFTER	
	●	○	●	○
+	3	2	3	2
x	1	3	1	3
Σ	8:2:0	9:3:3	8:2:0	7:2:1
∨	4	3	4	3

DIAGRAM 41. White's first turn: e7–e6, e8–e7, a3–a2, a2–a1.



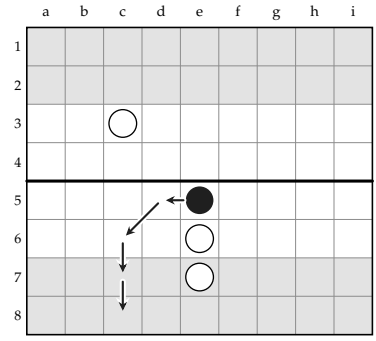
	BEFORE		AFTER	
	●	○	●	○
+	3	2	4	2
x	1	3	1	3
Σ	8:2:0	7:2:1	4:1:0	7:2:1
∨	4	3	4	3

DIAGRAM 42. Black's first turn: i5–i6, i6–i7, i7–i8, i8–off.



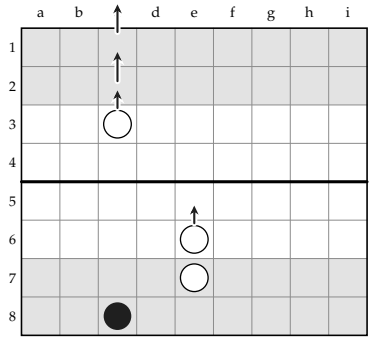
	BEFORE		AFTER	
	●	○	●	○
+	4	2	4	4
x	1	3	1	3
Σ	4:1:0	7:2:1	4:1:0	3:1:1
∨	4	3	4	3

DIAGRAM 43. White's second turn: a1–off, b3–b2, b2–b1, b1–off.



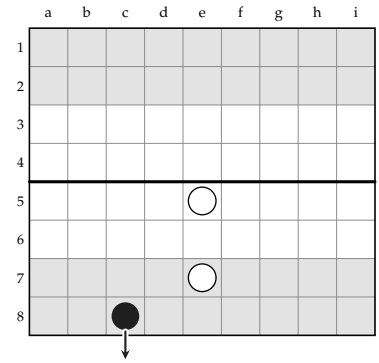
	BEFORE		AFTER	
	●	○	●	○
+	4	4	4	4
x	1	3	1	3
Σ	4:1:0	3:1:1	1:1:3	3:1:1
∨	4	3	1	3

DIAGRAM 44. Black's second turn: e5–d5, d5–c6, c6–c7, c7–c8.



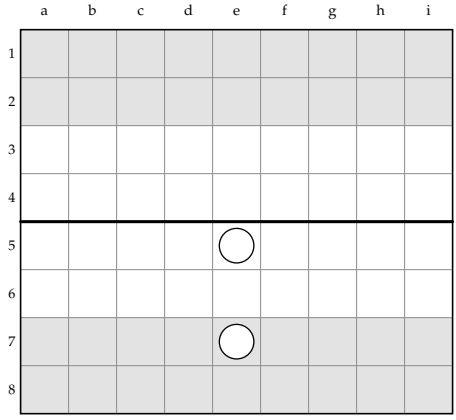
	BEFORE		AFTER	
	●	○	●	○
+	4	4	4	5
x	1	3	1	3
Σ	1:1:3	3:1:1	1:1:3	0:0:0
∨	1	3	1	–

DIAGRAM 45. White's third turn: c3–c2, c2–c1, c1–off, e6–e5.



	BEFORE		AFTER	
	●	○	●	○
+	4	5	5	5
x	1	3	1	3
Σ	1:1:3	0:0:0	0:0:0	0:0:0
∨	1	–	–	–

DIAGRAM 46. Black's third turn: c8–off.



	●	○
+	5	5
x	1	3
Σ	0:0:0	0:0:0
∨	–	–

DIAGRAM 47. The end. Five all. White has made three captures and Black one, and White wins.

After the block Black needs nine steps, which is three turns. White needs seven more, two turns. White reaches five on its third turn, before Black's runner is home.

The two plans used the same pearls and nearly the same number of steps. One removed a black pearl and the other only delayed it. The delay won and the capture lost, and the count says why before a step is played. Black had no spare steps, so one lost step was a turn. And Black's maximum was the target, so taking a pearl from Black lowered Black's count by four steps and White's by three.

22. The Main Points

1. A game is won by the side that reaches the score it needs in fewer turns. Everything else matters by how much it changes that.
2. Count in steps and compare in turns. A turn is four steps. Know both sides' spare steps.
3. The side to move wins equal turns. It can be behind by as many steps as the opponent has spare.
4. No arrangement gives an edge of more than one step in hand. Only arrangement D gives White the edge with a step to lose.
5. A sideways step loses one. A backward step loses two. A diagonal step forward loses nothing, so change file early.
6. The target is the lower maximum. Count only the pearls you need for it.
7. The side that reaches its meeting line first has to wait, and a side that waits loses four steps a turn. A level lock goes against the side that runs out of free steps first, which is usually the side that started ahead.
8. A row gained on one file changes the difference between the counts by two steps.
9. A pearl loses no steps to one or two blockers across its path. It loses two or three steps, once, to a solid line. A pair breaks a spaced line.
10. By a capture you gain the distance of your slowest counted pearl, less the distance of the pearl taken, less the steps you lose in making it. Take runners. Leave stragglers.
11. When the opponent has pearls off the count, taking one of them gains nothing.

12. Give up a pearl that is further from home than the opponent's slowest counted pearl, if you are level or behind in captures.
13. Chase with pearls at home or off the count. A counted pearl loses two steps for each backward step.
14. You can use your spare steps without needing another turn. Use them on blocks. The opponent needs another turn only once it has lost more steps than it has spare.
15. The side with the lower maximum must end the game, and any step that brings the end nearer helps it.

When to Capture

The side ahead, the side behind, and the count

The number of steps a side gains by a capture can be worked out from the count. Whether the side should make the capture also depends on which side it is. The side that is ahead on the count wins if nothing changes, and this article calls it the attacker. The side that is behind loses if nothing changes, and is called the defender. The attacker needs a capture to be safe. The defender needs it to be enough. Part One sets out the sum for a capture and the test each side applies to it. Parts Two to Four take the opening, the middlegame and the ending in turn, with positions for each side. Part Five collects the results in a table.

Part One
The Sum and the Two Sides

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1. The Question

A capture changes the count of both sides, and the changes can be read from the board before a step is played. *Arithmetic Margrites*, in this volume, set out the method. Its sections on the capture discuss the sum in detail and work an example through. This article takes that method as given and uses its terms.

A pearl's *distance* is the number of steps it needs to cross. A side's *maximum* is its score plus its pearls on the board, and the *target* is the lower of the two maximums. A side's *count* is the number of steps it needs to reach the target, using its nearest pearls. Pearls it does not need are *off the*

count. Its *turns* are its count divided by four and rounded up, and what is left over in the last turn is its *spare* steps. The side that needs fewer turns has the *edge*. If the turns are equal, the side to move has it.

The method gives the number of steps a side gains or loses by a capture. It does not settle whether to make one, because the same number of steps matters differently to different sides. Two steps can change the turns of a side that is one step behind. They change nothing for a side that is two turns ahead and would have to turn its pearls aside to make the capture. A capture that loses a step may be right for a side that is losing anyway and wrong for a side that is winning.

The question here is therefore narrower than how many steps a side gains by a capture. It is when a capture should be made: by which side, and at what stage of the game.

2. The Sum

Suppose you capture a pearl while the captures are level, or while you are ahead in captures. The opponent's maximum falls by one, and so does the target. Two counts change.

The opponent needed every pearl it had. Its count falls by the distance of the pearl you took. That distance is the *near figure*.

You need one crossing fewer. Your slowest counted pearl comes off the count, and your count falls by its distance. That distance is the *far figure*.

Any step of the capture that did not go forward was a step lost.

> *The steps a side gains by a capture are the far figure, less the near figure, less the steps it loses in making the capture.*

Steps are lost as follows.

1. A forward step, straight or diagonal, loses nothing.
2. A sideways step loses one.
3. A backward step loses two, because the row has to be made up again.
4. A step by a pearl that is off the count, or that the capture takes off the count, loses one whichever way it goes. That pearl does not have to make up the rows it gives.

The case is different when you are behind in captures. The opponent then has the higher maximum and has pearls off the count. A capture

lowers its maximum but leaves the target where it was. Your own count does not change. The opponent's count changes only if the pearl you took was one it was counting on. It must then bring up a spare pearl in its place, and its count rises by the difference between the two distances. If the pearl you took was already off the count, the capture has changed nothing.

In either case the result is in steps, and it has to be turned into turns before it means anything. A gain of two steps may change the turns of both sides, of one, or of neither.

3. The Side Ahead and the Side Behind

The side with the edge will win the race if both sides simply run. It is called the *attacker* here. It has no need to change anything, and its whole interest is to reach the target with the count as it stands. The other side is the *defender*. It loses the race as things are, and it has to change the count or lose the game.

These names describe what each side needs. They do not describe where the pearls stand or which side is making threats. A side with half its pearls at home may be the attacker in this sense, and a side with three pearls deep in the other half may be the defender.

The two sides put a capture to different tests.

The attacker's test. The attacker wins by running. It captures only if it can show that the capture is safe, and there are three things to show.

1. The pearl taken is nearer to crossing than the attacker's own slowest counted pearl. Otherwise the capture lowers the defender's count by more than the attacker's.

2. The steps lost are covered, either by what the sum gains or by the attacker's spare steps. A side that needs three turns with a count of ten will play twelve steps, and two of them are wasted whatever it does. Those two can be used on a capture without needing another turn. A third lost step adds a turn.

3. Nothing worse can be taken back.

If any of the three is in doubt, the attacker does not capture. It runs.

The defender's test. The defender has two ways to change the count. It can make the attacker lose steps, or it can capture. For the defender a capture has one thing to show: that the count is better after it than before, when the reply has been allowed for. If the gain is enough to change the turns, the defender has the edge. If it is not, the defender is nearer than it was.

The defender can also accept a risk that the attacker cannot. A side that loses by running gives up nothing by trying a capture that may fail.

> *The attacker asks whether a capture is safe. The defender asks whether it is enough.*

The names change hands. After any capture that stands, the count has to be made again, and the side that was the defender may now be the attacker. From that moment it puts its captures to the attacker's test. A defender that has won the edge with one capture has no reason to look for a second unless the second is safe.

4. Where the Capture Is Made

The near figure is the distance of the pearl taken, and a pearl's distance depends only on the row it stands on. A pearl that has not left its home rows is seven or eight steps from crossing. A pearl on the center is four or five. A pearl that has reached your home rows is one or two.

So a side gains one step more by the same capture for each row the pearl has come. The table gives the steps gained by a capture, before any steps lost in making it, for each far figure and for each part of the board on which the pearl is taken.

	The pearl is taken on			
Far figure	*its home rows*	*its third or fourth row*	*your fourth or third row*	*your home rows*
8	0 or 1	2 or 3	4 or 5	6 or 7
7	−1 or 0	1 or 2	3 or 4	5 or 6
6	−2 or −1	0 or 1	2 or 3	4 or 5
5	−3 or −2	−1 or 0	1 or 2	3 or 4
4	−4 or −3	−2 or −1	0 or 1	2 or 3

Read across, the table shows that a side gains most by capturing on its own home rows. A pearl taken on your home rows had made nearly all its steps, and the capture wipes them out. A pearl taken on its own home rows had nearly all its steps still to make, and the capture relieves its side of them.

Read down, it shows that a side gains most by capturing when it has a pearl still far back. The far figure belongs to the side making the capture.

Both points bear differently on the two sides. The attacker is the side whose pearls, added together, have less far to go. On the whole its pearls

are the advanced ones, and the enemy pearls within their reach are the ones near the enemy's home rows. The defender's case is the reverse. Its pearls are further back, and the enemy pearls that come within their reach have come a long way. The captures open to the defender tend to lie on the right of the table and in its upper rows. The captures open to the attacker tend to lie on the left and lower down.

The captures open to the defender gain more steps than the captures open to the attacker.

This is a tendency and not a law. It is weakest in the opening, when the two sides are nearly alike, and strongest in the ending.

5. What Is Taken Back

The attacker's third question needs a rule of its own. Suppose a capture is answered by a capture, so that each side has lost one pearl and the captures are level again. The target has fallen by one for both. Each side needs every pearl it has left. So each side's count has fallen by the distance of the pearl it lost, and by nothing else.

In an exchange each side's count falls by the distance of the pearl it lost. The exchange favors the side that lost the pearl further from crossing.

It sounds wrong that losing the further pearl should be the better half of an exchange, but it is the same point as before. The pearl far from crossing had most of its steps still to make. The pearl near to crossing had made them.

It follows that an exchange, like a single capture, is better at home. In your own half the pearl you lose is far from crossing and the pearl you take is near. In the opponent's half it is the other way round. A pearl that goes forward to capture and is captured in return has usually lost the exchange, although the numbers of pearls are equal.

When the reply takes two pearls for one there is no short rule. The count has to be made again from the position after the reply. Section 9 gives an example.

6. The Three Phases

The *opening* lasts until the two sides meet. The *middlegame* lasts while they are in contact. The *ending* begins when most of the pearls have passed each other and the count can be made exactly.

The sum is the same in all three. What changes is the size of its terms. In the opening both far figures are seven or eight and every pearl that can be taken is four or five steps from crossing, so a side gains about the same number of steps by a capture whichever side it is. In the middlegame the figures spread out. Pearls behind the lines are seven or eight steps from crossing, pearls in the lines five, and pearls that have broken through two or three. In the ending the figures are as far apart as they can be, and a capture may also end the game.

Each of the next three Parts takes the attacker and the defender separately. In every diagram the figure on a pearl is its distance. Arrows show the steps of the turn being played, and a cross marks a pearl that those steps capture.

Part Two
The Opening

7. A Side Gains Steps by Every Capture

Every pearl starts seven or eight steps from crossing. The far figure of both sides is therefore seven or eight, and it stays there for as long as one pearl has not left its home rows. The pearls that can be captured in the opening are the ones that have gone forward. They are taken on or near the center, four or five steps from crossing. So in the opening a side gains between two and four steps by a capture, before any steps it loses in making it, whichever side it is.

Against this, the edge in the opening is very small. In every starting arrangement the side with the edge has at most one step in hand, as the section of *Arithmetic Margrites* on the arrangements discusses. A side that makes the first capture of the game gains more steps by it, if it stands,

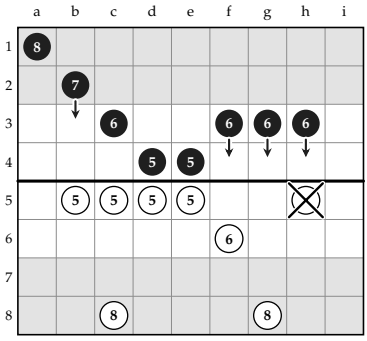
than the edge was. It usually decides which side is the attacker in the middlegame.

In the opening, then, the two sides are not yet very different. Neither is far ahead, both have the same far figure, and each gains about the same number of steps by the same capture. The questions of Section 3 still apply, but the first of them nearly always has the same answer: the pearl is nearer to crossing than the slowest pearl, and the sum is positive. What is left to decide is the steps lost in making the capture and the reply. The defender has to look mainly at the lost steps. The attacker has to look mainly at the reply.

8. The Defender: The First Capture

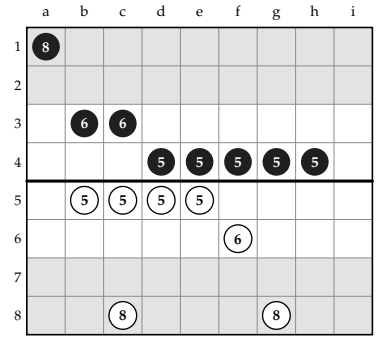
The position of Diagram 1 arose from arrangement D, in which White places five pearls forward and Black three. Each side has played three turns and neither has lost a step. Black is to play. White's count is 47 and Black's is 49, so White needs twelve turns and Black thirteen. White is the attacker, with one step in hand.

White's pearl on h5 has gone to the center line by itself, and no white pearl stands next to it. The black pearls on g3 and h3 can each step forward and stand beside it.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	1	0
Σ	49:13:3	47:12:1	37:10:3	42:11:2
∨	8	8	6	8

DIAGRAM 1. Black to play. White has the edge. Black plays g3–g4, h3–h4, f3–f4, b2–b3.



	●	○
+	0	0
x	1	0
Σ	37:10:3	42:11:2
∨	6	8

DIAGRAM 2. After Black's turn. The pearl on a1 is off the count.

		Black		*White*		*To*	
	Target	*count*	*turns*	*count*	*turns*	*move*	*Edge*
Before	8	49	13	47	12	Black	White
After	7	37	10	42	11	White	Black

Black plays g3–g4, h3–h4, f3–f4, b2–b3. The first two steps capture the pearl on h5.

The near figure is five, the distance of the pearl taken. The far figure is eight: Black's slowest pearl is the one on a1, which has not moved. Both capturing steps went forward and lost nothing. Black gains eight less five, three steps.

The target is now seven. Black's count is 37, ten turns. White's is 42, eleven turns, and it is White's move. Before the capture White was a turn ahead. After it Black is a turn ahead, with three steps in hand.

Three things about this capture hold for the defender's captures in the opening generally.

First, Black could have lost more steps in making it. Black had three spare steps before the capture. Had one of the capturing steps been sideways, or both of them, Black would have finished the turn with a count of 38 or 39, and that is still ten turns. The defender in the opening nearly always has spare steps, and a capture is the best use for them.

Second, the far figure was the pearl on a1. That pearl has not moved in the game, and now it does not have to. A defender that means to capture should be in no hurry to bring its last pearl off the back row. While other pearls have forward steps to play, nothing is lost by leaving it, and a pearl on the back row adds a step more to the sum than a pearl on the row in front.

Third, Black's other two steps were played by pearls that are on the count. A step by the pearl on a1 would now be a step lost. After a first capture the side that made it has a pearl it does not need, and it has to remember which pearl that is.

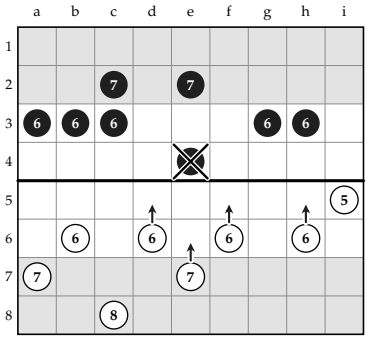
The capture has also changed the names. Black is now the attacker and White the defender, and from here Black asks the attacker's questions.

9. The Attacker: The Offered Pearl

Diagram 3 is from the same arrangement, a turn earlier and with the pearls differently placed. Black has played three turns and White two, and White is to play. White's count is 51 and Black's 49. Both need thirteen turns and White moves first, so White is the attacker.

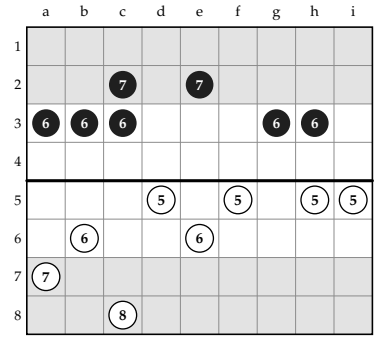
Black has a pearl on e4 with no black pearl next to it. The white pearls on d6 and f6 can each step forward and stand beside it. White loses no steps in making the capture. The near figure is five, and the far figure is eight, for the white pearl still on c8. The sum is three steps, as in the last position.

That answers the attacker's first two questions. The third is what can be taken back, and the answer depends on what White does with the other two steps of the turn.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	1
Σ	49:13:3	51:13:1	44:11:0	39:10:1
∨	7	8	7	7

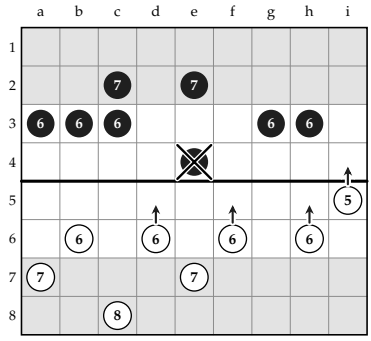
DIAGRAM 3. White to play. First line: d6–d5, f6–f5, e7–e6, h6–h5.



	●	○
+	0	0
x	0	1
Σ	44:11:0	39:10:1
∨	7	7

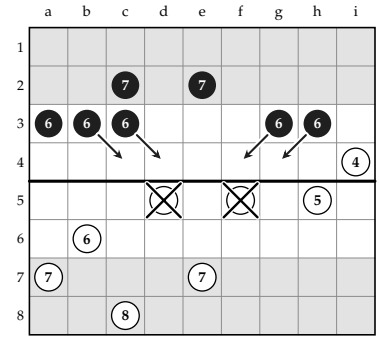
DIAGRAM 4. After the first line. The pearl on e6 stands beside both capturing pearls.

In the first line White plays d6–d5, f6–f5, e7–e6, h6–h5. The third step brings a pearl to e6, where it stands next to both of the pearls that made the capture. Each of those now has a white pearl beside it, and it would take four black pearls round either one to capture it. Black cannot bring four. The capture stands. The target is seven, and White needs ten turns to Black's eleven. Black had three spare steps before the capture and now has none. Before the capture a gain of one step would have changed the turns in Black's favor. Now it would take four.



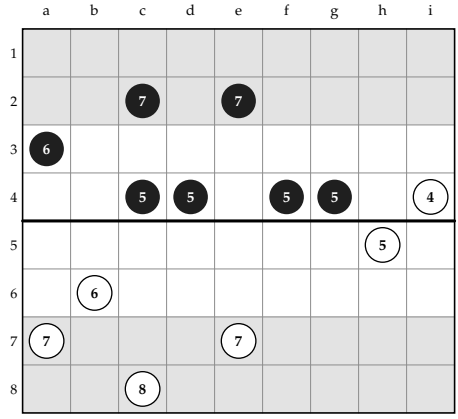
	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	1
Σ	49:13:3	51:13:1	44:11:0	39:10:1
∨	7	8	7	7

DIAGRAM 5. Second line. White makes the same capture and plays d6–d5, f6–f5, h6–h5, i5–i4.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	1	2	1
Σ	44:11:0	39:10:1	33:9:3	37:10:3
∨	7	7	7	8

DIAGRAM 6. Black replies b3–c4, c3–d4, h3–g4, g3–f4 and takes both pearls.



	●	○
+	0	0
x	2	1
Σ	33:9:3	37:10:3
∨	7	8

DIAGRAM 7. After Black's reply. White has six pearls and Black seven.

In the second line White makes the same capture and plays its other two steps on the right: d6–d5, f6–f5, h6–h5, i5–i4. The counts after White's turn are the same as in the first line, 39 and 44. The pearls on d5 and f5 now each stand alone, and Black has two pearls within a step of each. Black

plays b3–c4, c3–d4, h3–g4, g3–f4. The first two steps capture the pearl on d5 and the last two the pearl on f5. All four go forward.

The count has to be made again. White has six pearls and Black seven, so the target is six. White needs every pearl it has left, and its count is 37, ten turns. Black needs six crossings from seven pearls. One of the pearls on its second row comes off the count, and Black's count is 33, nine turns. It is White's move, and Black is a turn ahead.

		Black		*White*		*To*	
	Target	*count*	*turns*	*count*	*turns*	*move*	*Edge*
Before	8	49	13	51	13	White	White
First line	7	44	11	39	10	Black	White
Second line	6	33	9	37	10	White	Black

White made the same capture in both lines and gained the same three steps by it. In the first line it kept them. In the second it lost two pearls in its opponent's half and gave Black a pearl off the count.

The difference between the lines is one step, e7–e6, and that step went forward. It lost nothing on the count. This is the usual case in the opening. The pearl that makes a capture safe can nearly always be brought up by a forward step, straight or diagonal, and when it can the attacker has no reason to leave it behind. When it cannot, the attacker should treat the pearl on offer as the defender's capture and not its own.

Black's part in this deserves a word. Black was the defender, and it left a pearl where it could be taken. If White answered correctly Black stood to lose three steps and be a turn behind. If White answered carelessly Black stood to be a turn ahead. Black was losing the race in any case. That is the kind of risk Section 3 allows to the defender and not to the attacker.

10. The Opening in Brief

For the attacker. A side gains about three steps by taking a pearl that stands alone on the center, and the attacker should take it if the pearls that take it will not stand alone afterwards. The attacker counts the reply before making the capture.

For the defender. The defender should take the same pearl even if it loses as many steps as it has spare in making the capture. The back pearl stays at home, and after the capture it takes no more steps.

For both. A side that makes the first capture that stands gains more steps than the edge. The side that makes it is the attacker from then on.

Part Three

The Middlegame

11. The Near and Far Figures in the Middlegame

When the two sides are in contact the pearls on the board are of three kinds. There are the pearls in the lines, on or beside the center, about five steps from crossing. There are the pearls left behind the lines on the home rows, seven or eight steps from crossing. And sooner or later there are pearls that have gone through a line or round it, two to four steps from crossing.

The sum treats the three kinds very differently, and the table in Section 4 can be read for each.

A side gains by taking a pearl in a line for as long as it has a pearl at home: eight or seven, less five. A side with every pearl in the line or beyond it gains five less five, which is nothing.

A side gains almost nothing by taking a pearl behind a line, and most sides lose steps by it.

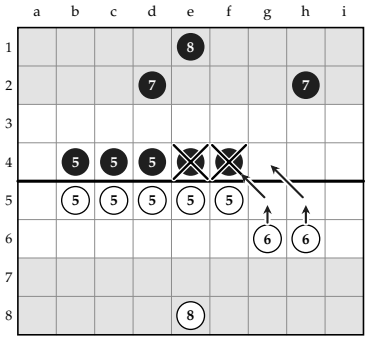
A side gains more by taking a pearl that has come through for every row the pearl has come, and most of all on its own home rows.

The attacker's captures are of the first two kinds. They are made at the enemy line or behind it. The defender's are of the first kind and the third. The next four sections take them in that order.

12. The Attacker: The End of the Line

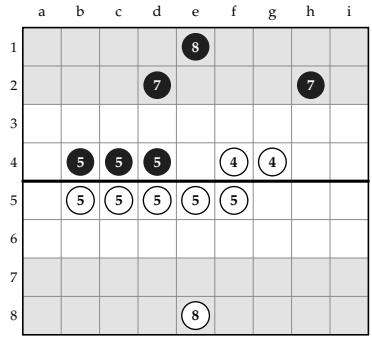
In Diagram 8 two lines of five face each other across the center. White has two more pearls coming up on the right and one at home. Black has three at home. White's count is 45 and Black's 47. Each side needs twelve turns, and White is to move. White is the attacker, with three steps in hand.

The black pearl on f4 is the end of its line. It has one black pearl beside it, so it takes four white pearls to capture it, and two stand next to it already. The two white pearls on the right can supply the others.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	2
Σ	47:12:1	45:12:3	37:10:3	28:7:0
∨	8	8	8	5

DIAGRAM 8. White to play: g6–g5, h6–h5, h5–g4, g5–f4.



	●	○
+	0	0
x	0	2
Σ	37:10:3	28:7:0
∨	8	5

DIAGRAM 9. After White's turn. Two pearls taken with four forward steps.

		Black		*White*		*To*	
	Target	*count*	*turns*	*count*	*turns*	*move*	*Edge*
Before	8	47	12	45	12	White	White
After	6	37	10	28	7	Black	White

White plays g6–g5, h6–h5, h5–g4, g5–f4. After the third step there are white pearls on e5, f5, g5 and g4 round the pearl on f4, and it is captured. The fourth step puts a white pearl on the square it stood on. That gives the pearl on e4 four white pearls beside it, on d5, e5, f5 and f4, against one black one, and it is captured as well. All four steps went forward.

We count the two captures one at a time.

For the first, the near figure is five. The far figure is eight, for the pearl on e8. White gains three steps by it.

For the second, the near figure is again five. White's slowest pearl is now off the count, and the next slowest is one of the pearls in the line, at five. White gains five less five, which is nothing.

The count agrees. The target is six. White needs six of its eight pearls, and the pearl on e8 and one pearl of the line are off the count: 28 steps, seven turns. Black needs all six of its pearls: 37 steps, ten turns. Before the turn each side needed twelve.

Two points follow for the attacker.

White gained its three steps by the first capture because the pearl on e8 was still at home. An attacker that has brought its last pearl up into the line has a far figure of five, and such a side gains nothing on the sum by

taking the pearl on the end of the enemy line. The attacker gains steps by its captures at the line because it has a pearl left at home. It loses nothing by leaving that pearl there while other pearls have forward steps to play.

White gained nothing on the sum by the second capture, and the capture was right all the same. White lost no steps in making it, the pearl taken was no further from crossing than White's slowest, and nothing can be taken back, since every white pearl near the black line has a white pearl beside it. It passes the attacker's test. White gains something that the sum does not show. Black's line is two pearls shorter and two files are open, and White's pearls will not now lose steps in front of that line.

13. The Attacker: The Pearl Left at Home

Diagram 10 is a later stage of a game. Each side has lost two pearls and neither has scored. The two sides have gone past each other on opposite wings, and nothing is in contact except one black pearl that never left h1. White's count is 24 and Black's 21. Each needs six turns, and White is to move. White is the attacker, with nothing in hand.

The white pearls on g3 and h3 are on their way to cross. Each can step forward to a square next to the pearl on h1, and the two steps capture it. They lose nothing, and the pearls that make them go on to cross as they would have done anyway.

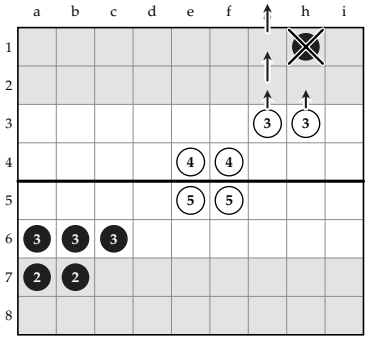
		Black		*White*		*To*	
	Target	*count*	*turns*	*count*	*turns*	*move*	*Edge*
Before	6	21	6	24	6	White	White
First line	5	13	4	15	4	Black	Black
Second line	6	21	6	20	5	Black	White

In the first line White plays g3–g2, h3–h2, g2–g1, g1–off, and the second step captures the pearl. The near figure is eight. White's slowest pearls are on its fifth row, so the far figure is five. White gains five less eight, which means that it loses three steps.

After it the target is five. White has scored one and needs four more crossings: 15 steps, four turns. Black needs all five of its pearls: 13 steps, four turns. It is Black's move, and Black has the edge.

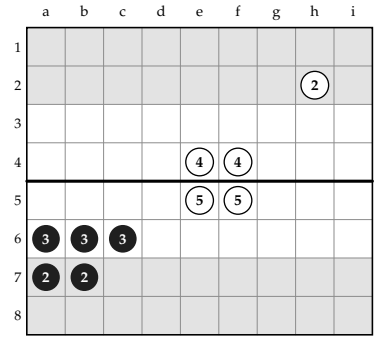
In the second line White runs with g3–g2, g2–g1, g1–off, h3–h2 and leaves the pearl on h1 where it is. White's count is 20, five turns, and Black's is 21, six turns. White still has the edge.

Eight of Black's 21 steps belonged to the pearl on h1. It was the reason Black was behind. White took it off the board with two steps that lost



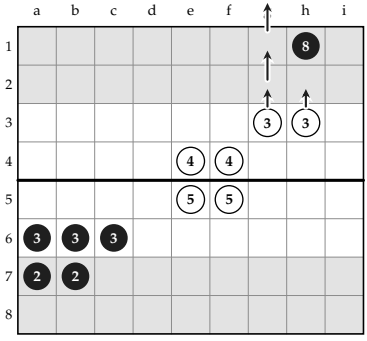
	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	1
x	2	2	2	3
Σ	21:6:3	24:6:0	13:4:3	15:4:1
∨	8	5	3	5

DIAGRAM 10. First line. White takes the pearl on h1: g3–g2, h3–h2, g2–g1, g1–off.



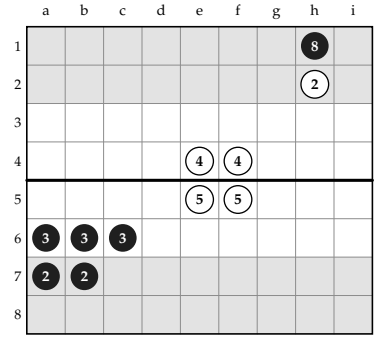
	●	○
+	0	1
x	2	3
Σ	13:4:3	15:4:1
∨	3	5

DIAGRAM 11. After the first line. White has scored one. Black is to move and has the edge.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	1
x	2	2	2	2
Σ	21:6:3	24:6:0	21:6:3	20:5:0
∨	8	5	8	5

DIAGRAM 12. Second line. White runs: g3–g2, g2–g1, g1–off, h3–h2.



	●	○
+	0	1
x	2	2
Σ	21:6:3	20:5:0
∨	8	5

DIAGRAM 13. After the second line. White has scored one and keeps the edge.

nothing, and in doing so cut the turns Black needs by two and its own by one.

This is the bottom left corner of the table in Section 4, and it is where an attacker's captures are most often found. The pearls an advancing side passes are the ones its opponent left behind, and those are the pearls

furthest from crossing. They are also the easiest to take. They stand alone, and the pearls passing them are going that way already.

> *The attacker leaves alone any pearl that is further from crossing than its own slowest pearl, however few steps it would lose in taking it, even none.*

The defender can make use of this. A pearl left at home in the attacker's path is no weakness on the count. If it is taken, the defender gains by it.

The attacker meets the same question from the other direction when a defender's pearl gets through behind it. To take that pearl the attacker's pearls have to turn round. They are pearls on the count, each backward step loses two, and the pearls that go back become the slowest the attacker has. The rule of Section 15 applies to both sides, and an attacker with its pearls well forward will seldom meet it. In most cases the attacker should let the pearl go and count the race.

14. The Defender: Let It Come

The defender's captures in the middlegame are of pearls that have come through its line or round it. By the table in Section 4, a side gains a step more by taking such a pearl for every row it comes. On the center it gains what it would gain by taking any pearl in a line. On the defender's home rows a side whose slowest pearl is at eight gains six or seven steps by taking it.

There is therefore no hurry to take it. A pearl that has broken through is going to come nearer, and the defender's pearls nearest to it when it reaches the defender's home rows are the ones that never left home. They are also the pearls that lose the fewest steps in capturing.

The pearl that captures	*Steps lost by each step*
A home pearl stepping forward	nothing
A home pearl stepping sideways	one
The pearl that the capture takes off the count	one, either way
A pearl from further forward stepping back	two

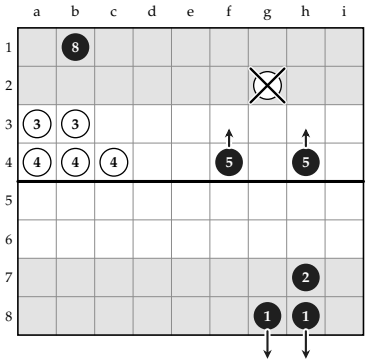
The first three lines are the defender's ordinary captures. With those losses the defender nearly always gains a turn by taking a pearl on its home rows. The pearl in the last line loses the most steps, and Section 15 discusses it.

The defender should also notice what it is that lets the home pearls do this work. They are far from crossing, which makes the far figure large. And they stand where the attacker's pearls have to pass. A defender that brings every pearl up to the line gives away both.

15. The Defender: Going Back

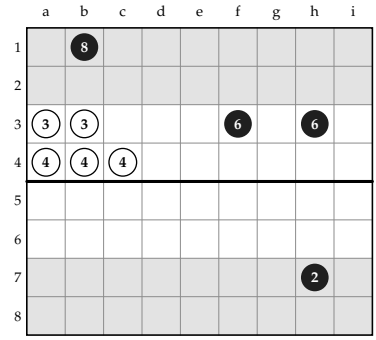
In Diagram 14 each side has lost two pearls and neither has scored. White has gone through on the left with five pearls and Black on the right with three. Black has a pearl at home on b1, and two on its fourth row that the game has left behind. A white pearl has passed between those two and stands on g2. Black is to play. Black's count is 22, six turns. White's is 20, five turns. White is the attacker.

No black pearl is near enough to take the pearl on g2 by going forward or sideways. The pearls on f4 and h4 can each take one step back and stand beside it.



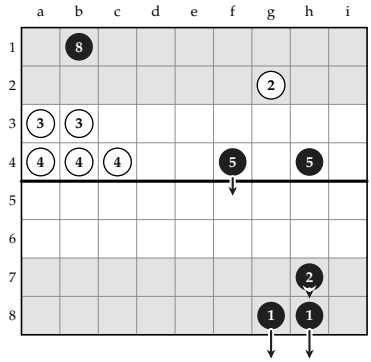
	BEFORE		AFTER	
	●	○	●	○
+	0	0	2	0
x	2	2	3	2
Σ	22:6:2	20:5:0	14:4:2	18:5:2
∨	8	4	6	4

DIAGRAM 14. First line. Black goes back for the pearl on g2: f4–f3, h4–h3, g8–off, h8–off.



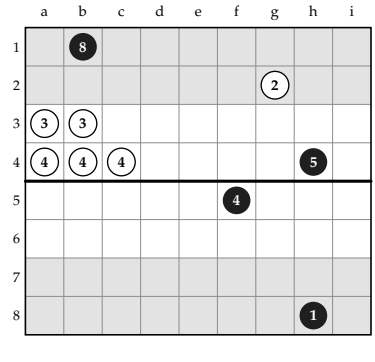
	●	○
+	2	0
x	3	2
Σ	14:4:2	18:5:2
∨	6	4

DIAGRAM 15. After the first line. Black has scored two. The pearl on b1 is off the count.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	2	0
x	2	2	2	2
Σ	22:6:2	20:5:0	18:5:2	20:5:0
∨	8	4	8	4

DIAGRAM 16. Second line. Black lets it go and runs: g8–off, h8–off, h7–h8, f4–f5.



	●	○
+	2	0
x	2	2
Σ	18:5:2	20:5:0
∨	8	4

DIAGRAM 17. After the second line. Black has scored two. White is to move and keeps the edge.

		Black		*White*		*To*	
	Target	*count*	*turns*	*count*	*turns*	*move*	*Edge*
Before	6	22	6	20	5	Black	White
First line	5	14	4	18	5	White	Black
Second line	6	18	5	20	5	White	White

In the first line Black plays f4–f3, h4–h3, g8–off, h8–off. The first two steps go backward and capture the pearl on g2. The other two cross the pearls on g8 and h8.

The near figure is two. The far figure is eight, for the pearl on b1, which comes off the count. Two backward steps by pearls on the count lose four. The sum is eight, less two, less four: two steps.

After it the target is five. Black has scored two and needs three more crossings, from pearls two, six and six steps away: 14 steps, four turns. White needs all five of its pearls: 18 steps, five turns. It is White's move, and Black has the edge. White's pearls can take the pearl on b1 as they go by, and they gain nothing if they do, because that pearl is off the count.

In the second line Black lets the pearl go and runs with g8–off, h8–off, h7–h8, f4–f5. Black's count is 18 and White's 20. Both need five turns and White moves first. White keeps the edge.

The backward capture was right here, by two steps. Those two steps depended on three things.

They depended on the pearl on b1. Suppose Black's slowest pearl had been one of the two that went back. The capture would then have taken that pearl off the count. Its backward step would have lost one step and the other pearl's two, and the far figure would have been five. The sum would have been five, less two, less three: nothing.

They depended on the pearls having only one row to go back. Each further row would have lost two steps for each pearl, and the whole gain was two.

And they depended on there being nothing near the squares the two pearls left. When the pearl that goes back comes out of a line, the sum does not show everything the defender loses. The square it leaves is open to the enemy pearl opposite, and the pearls that stood beside it have one supporting pearl fewer. The defender finds that loss as it finds the rest, by asking what the attacker can now take and how many steps the attacker would gain by it. A defender that gains two steps by a backward capture out of a line, and lets the attacker gain three by a capture in reply, has lost one.

> *Go back for a pearl only when your slowest pearl is further from crossing than the pearl you are going back for, by more than twice the backward steps.*

16. The Middlegame in Brief

For the attacker. Capture at the enemy line while a pearl remains at home to give a large far figure. Leave the pearls behind the line alone, and do not turn round for a pearl that has got past.

For the defender. Let a pearl that has broken through come on, and take it with the pearls at home. Go back for it only under the rule of Section 15.

Part Four
The Ending

17. Exact Figures

In the ending few pearls are left and most of them have passed each other. Both counts can be made to the last step, and the sum can be trusted in a way that it cannot be earlier.

The figures in it are also as far apart as they can be. A pearl about to cross is one or two steps from crossing, and a pearl that was left at home is seven or eight. A capture can gain six steps or lose six, and either is more than a turn.

Two things are new.

The first is that the captures are often no longer level. The second case of Section 2 then comes up: a side that is behind in captures gains only by taking pearls that are on the count.

The second is the end of the game itself. The game is over when one side has no pearls on the board. A capture can bring that about. It can also bring the target down to a score that has already been reached. In both cases the sum is beside the point, and Sections 19 and 21 take them.

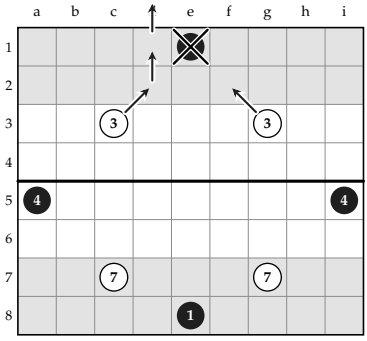
18. The Attacker: A Capture That Loses Steps and a Capture That Loses None

In Diagram 18 each side has scored two and lost two, and has four pearls left. White's count is 20 and Black's 17. Each needs five turns, and White is to move. White is the attacker, with nothing in hand.

Two captures are open to White without turning a pearl round.

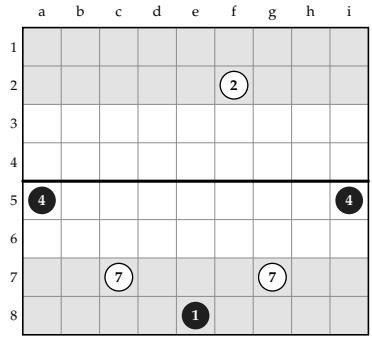
The black pearl on e1 has not moved in the game. White's two advanced pearls can each step forward to a square beside it, d2 and f2, and it is captured. White loses no steps in making the capture.

The black pearl on e8 is one step from crossing. It stands a row behind White's two home pearls and between them. If each of them steps sideways, to d7 and f7, it is captured. White loses two steps in making the capture.



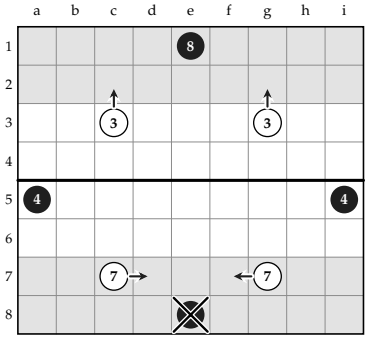
	BEFORE		AFTER	
	●	○	●	○
+	2	2	2	3
x	2	2	2	3
Σ	17:5:3	20:5:0	9:3:3	9:3:3
∨	8	7	4	7

DIAGRAM 18. The capture with no lost steps: c3–d2, g3–f2, d2–d1, d1–off.



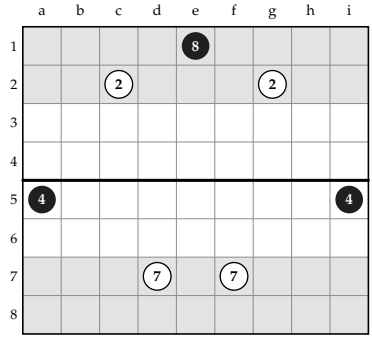
	●	○
+	2	3
x	2	3
Σ	9:3:3	9:3:3
∨	4	7

DIAGRAM 19. After it. White has scored three. Black is to move and has the edge.



	BEFORE		AFTER	
	●	○	●	○
+	2	2	2	2
x	2	2	2	3
Σ	17:5:3	20:5:0	16:4:0	11:3:1
∨	8	7	8	7

DIAGRAM 20. The capture that loses two steps: c7–d7, g7–f7, c3–c2, g3–g2.



	●	○
+	2	2
x	2	3
Σ	16:4:0	11:3:1
∨	8	7

DIAGRAM 21. After it. White keeps the edge and has a step in hand.

		Black		*White*		*To*	
	Target	*count*	*turns*	*count*	*turns*	*move*	*Edge*
Before	6	17	5	20	5	White	White
Taking e1	5	9	3	9	3	Black	Black
Taking e8	5	16	4	11	3	Black	White
Running	6	17	5	16	4	Black	White

Take first the capture with no lost steps. White plays c3–d2, g3–f2, d2–d1, d1–off. The near figure is eight and the far figure seven, so White loses a step by the capture. After it the target is five. White has scored three and needs two more crossings: nine steps, three turns. Black needs its three pearls home: nine steps, three turns. Black is to move and has the edge.

Now the capture that loses two steps. White plays c7–d7, g7–f7, c3–c2, g3–g2. The near figure is one. The far figure is seven, since White now needs one home pearl fewer. The lost steps are two. The sum is seven, less one, less two: four steps. After it White needs three turns and Black four. White keeps the edge and has a step in hand.

If White makes neither capture and runs, it needs four turns to Black's five, and still has nothing in hand.

The attacker's third question has still to be asked of the capture on e8. Black can take back one of the white pearls that now stand on Black's second row. To do it Black has to step the pearl on e1 sideways and bring a pearl back two rows from its fifth row. After that White needs four turns and Black five. Black loses more steps by the reply than it gains, and the capture stands.

The capture that lost two steps was right, and the capture that lost none was wrong. In the ending the lost steps are the smallest of the three terms. The near figure and the far figure can each be anything from one to eight, and an attacker that looks first at the steps it will lose is looking at the wrong term.

White also lost the two steps without having any to spare. Its count before the capture was five turns exactly. The steps gained by the sum covered the lost steps, which is the first of the two ways that Section 3 allows.

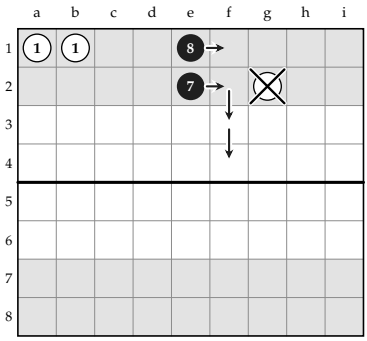
A third capture was possible. White could have turned one of its advanced pearls back and taken a pearl on Black's fifth row. By it White would gain seven, less four, less two for the backward step: one step. That is less than White gains by the capture on e8, and it leaves the pearl on e8 to cross.

> *In the ending the attacker takes the near pearl, even if it loses steps in doing so, and leaves the far pearl, even if it would lose none.*

19. The Defender: When the Target Falls to Your Score

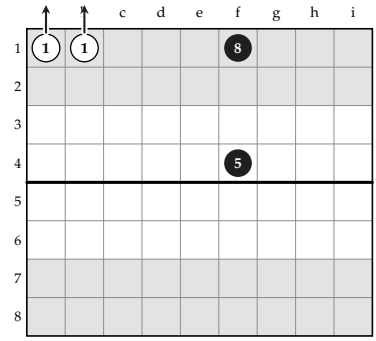
In Diagram 22 Black has scored four, has lost two pearls, and has two left, both at home. White has scored two, has lost three, and has three left, all about to cross. Black's maximum is six and White's five, so the target is five. Black needs one more crossing, and its nearer pearl is seven steps away: two turns. White needs all three of its pearls, four steps in all: one turn. Black is to play. White is the attacker, and if Black does nothing to stop it White crosses all three pearls next turn and wins by five to four.

The white pearl on g2 stands alone. Black's two pearls can each step sideways and stand beside it.



	BEFORE		AFTER	
	●	○	●	○
+	4	2	4	2
x	3	2	4	2
Σ	7:2:1	4:1:0	0:0:0	2:1:2
∨	7	2	–	1

DIAGRAM 22. Black has scored 4 and White 2. Black plays e1–f1, e2–f2, f2–f3, f3–f4.



	BEFORE		AFTER	
	●	○	●	○
+	4	2	4	4
x	4	2	4	2
Σ	0:0:0	2:1:2	0:0:0	0:0:0
∨	–	1	–	–

DIAGRAM 23. The target is four and Black has four. White plays a1–off, b1–off and the game ends.

Black plays e1–f1, e2–f2, f2–f3, f3–f4. The first two steps capture the pearl on g2.

By the sum Black gains seven, less two, less two: three steps. The sum does not decide this position. White's maximum is now four, so the target is four, and Black has four already. Black needs nothing more.

White can still cross its two pearls, and does, with a1–off, b1–off. The score is four all and White has no pearls on the board, so the game is over. Black has captured four pearls and White two. Black wins on captures.

A side that has reached the target cannot lose. The other side can at most draw level on the score, and it loses the tie, since the side with the higher maximum is the side that has made more captures.

A side wins the game by a capture that brings the target down to its score.

The side most likely to be able to use this is the defender. A side can be well ahead on the score and still behind on the count: its quick pearls have crossed and its slow ones are all it has left. That side cannot win the race, and it does not need to. It needs as many captures as the opponent's maximum exceeds its own score. In Diagram 22 that was one.

The same figure tells the attacker what it has to prevent. An attacker whose maximum is one above the defender's score must not leave a pearl alone within reach of two of the defender's pearls, however near to crossing that pearl is.

20. The Defender: Behind in Captures

In Diagram 24 Black has scored three and lost three pearls. White has scored two and lost one. Black's maximum is five and White's seven, so the target is five. Black needs both its pearls home: 15 steps, four turns. White needs three more crossings and has five pearls to make them with. It counts the nearest three, which are two, four and four steps from crossing: ten steps, three turns. The two pearls on its home rows are off the count. Black is to play, and White is the attacker.

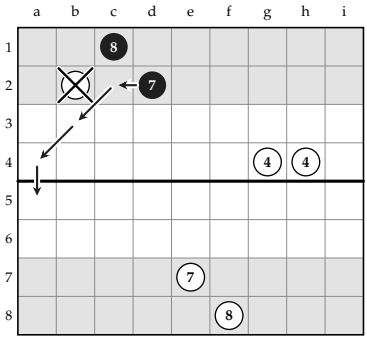
The white pearl on b2 has a black pearl next to it already, on c1. If the pearl on d2 steps sideways to c2, it is captured.

		Black		*White*		*To*	
	Target	*count*	*turns*	*count*	*turns*	*move*	*Edge*
Before	5	15	4	10	3	Black	White
After the capture	5	12	3	15	4	White	Black
Running	5	11	3	10	3	White	White

Black plays d2–c2, c2–b3, b3–a4, a4–a5. The first step captures and the other three go forward.

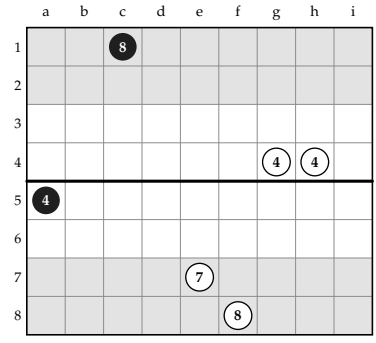
This is the second case of Section 2. White had the higher maximum, and it still has: six against five. The target has not moved, and Black needs exactly what it needed before. Its count has changed only by the steps it played, three forward and one lost.

White's count has changed. The pearl on b2 was one of the three it was counting on. A spare pearl has to take its place, and the nearer of the two is on e7, seven steps from crossing. White's count rises by seven less



	BEFORE		AFTER	
	●	○	●	○
+	3	2	3	2
x	1	3	2	3
Σ	15:4:1	10:3:2	12:3:0	15:4:1
∨	8	4	8	7

DIAGRAM 24. Black has scored 3 and White 2. Black plays d2–c2, c2–b3, b3–a4, a4–a5.



	●	○
+	3	2
x	2	3
Σ	12:3:0	15:4:1
∨	8	7

DIAGRAM 25. After Black's turn. White must now count the pearl on e7.

two, five steps, to 15. White now needs four turns and Black three. Black has the edge. Had Black run instead, both sides would have needed three turns with White to move.

For the side behind in captures the sum has the same shape as before, but the far figure is the opponent's. It is the distance of the spare pearl that has to replace the pearl taken.

> *A side behind in captures gains by a capture the distance of the opponent's nearest spare pearl, less the distance of the pearl taken, less the steps lost. It gains nothing by taking a pearl that was off the count.*

Three things follow for the defender.

It should take the nearest pearl it can reach. The far figure is fixed by the opponent's spare pearls, so the whole of the difference lies in the near figure. Had the pearl taken been one of those four steps from crossing, White's count would have risen by three and not by five.

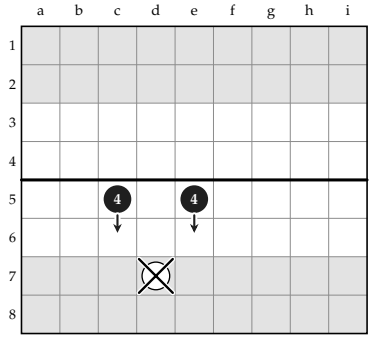
It should not use a step on a pearl that is off the count. Such pearls are often the easiest to reach, since they are the ones standing still.

And it should know how far a series of captures has to go. White has one spare pearl left. A second capture of a counted pearl would bring that one onto the count as well. Only a third would lower the target.

21. Either Side: The Last Pearl

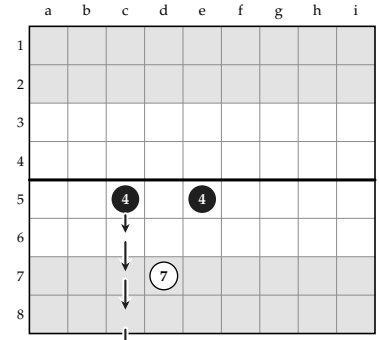
In Diagram 26 Black has scored three and has two pearls left. White has scored four and has one. Each side has lost three. The maximums are equal at five, and each side needs every pearl it has. Black's count is eight and White's seven. Both need two turns and Black is to move, so Black is the attacker.

White's last pearl stands on d7, between the files that Black's two pearls are coming down. One forward step by each of them puts a black pearl on either side of it.



	BEFORE		AFTER	
	●	○	●	○
+	3	4	3	4
x	3	3	4	3
Σ	8:2:0	7:2:1	3:1:1	0:0:0
∨	4	7	3	–

DIAGRAM 26. Black has scored 3 and White 4. The capture, c5–c6, e5–e6, ends the game. White wins.



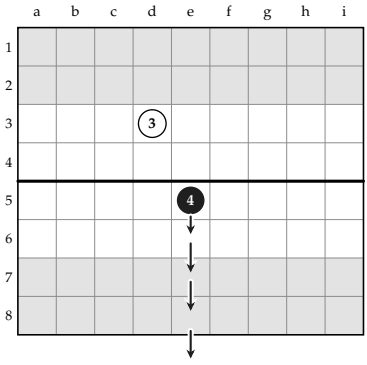
	BEFORE		AFTER	
	●	○	●	○
+	3	4	4	4
x	3	3	3	3
Σ	8:2:0	7:2:1	4:1:0	7:2:1
∨	4	7	4	7

DIAGRAM 27. The right first turn: c5–c6, c6–c7, c7–c8, c8–off. Black has four.

If Black plays c5–c6, e5–e6, the pearl on d7 is captured with the second step. White has no pearls on the board and the game is over. The score is three to four, and White has won.

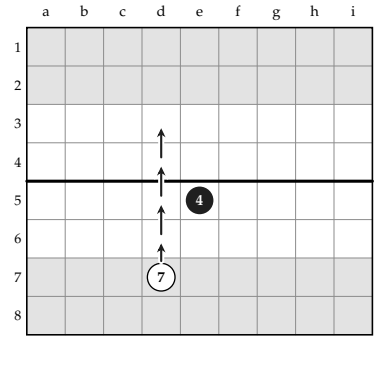
Black would lose no steps in making the capture. The capture is also the last thing White needs. A capture takes a pearl off the opponent's board and a crossing off the opponent's needs at once. When the pearl is the opponent's last, so is the crossing.

The right play is to go past. Black plays c5–c6, c6–c7, c7–c8, c8–off, taking one pearl down the c-file while the other stays out of reach of d7. White runs with its pearl, d7–d6, d6–d5, d5–d4, d4–d3. Black then plays e5–e6, e6–e7, e7–e8, e8–off, and has five with no pearls left on the board. White's pearl is still three steps from crossing. Black wins by five to four.



	BEFORE ●	BEFORE ○	AFTER ●	AFTER ○
+	4	4	5	4
x	3	3	3	3
Σ	4:1:0	3:1:1	0:0:0	3:1:1
∨	4	3	–	3

DIAGRAM 28. Black plays e5–e6, e6–e7, e7–e8, e8–off and wins by five to four.



	BEFORE ●	BEFORE ○	AFTER ●	AFTER ○
+	4	4	4	4
x	3	3	3	3
Σ	4:1:0	7:2:1	4:1:0	3:1:1
∨	4	7	4	3

DIAGRAM 29. White runs: d7–d6, d6–d5, d5–d4, d4–d3.

Before taking a side's last pearl, compare the scores. The game ends on that step.

The rule is the same for both sides, and the mistake is easy to make. The pearl is in the way, the capturing side loses no steps, and each step of it is one the capturing pearls meant to take anyway. The safeguard is to count the crossings each side needs before the turn begins.

22. The Ending in Brief

For the attacker. Take the near pearl and leave the far one. When your maximum is one above the defender's score, leave no pearl alone within reach of two of the defender's.

For the defender. Count the captures that bring the target down to your score. Behind in captures, take only counted pearls, the nearest first.

For both. Compare the scores before taking a last pearl.

Part Five

In Brief

23. The Table

	The attacker	*The defender*
Opening	Take the pearl that stands alone, and bring up the pearl that makes the capture safe. Count the reply first.	Take the pearl that stands alone if you lose no more steps than you have spare. Leave the back pearl at home.
Middlegame	Capture at the line while a pearl is at home to give a large far figure. Leave the pearls behind the line. Do not turn round.	Let the pearl come and take it with the home pearls. Go back only for more than twice the backward steps.
Ending	Take the near pearl, even if you lose steps. Leave the far pearl, even if you would lose none.	Count the captures that bring the target to your score. Behind in captures, take only counted pearls.
Throughout	A doubtful capture is not made.	A capture that leaves the count better is made.

Two rules belong to neither column. Before taking a side's last pearl, compare the scores. And after any capture that stands, count again and ask which side you now are.

24. The Questions in Order

Before a capture, the side with the edge asks four questions. 1. Is the pearl nearer to crossing than my slowest counted pearl? 2. Are the lost steps covered, by the sum or by my spare steps? 3. What can be taken back, and what is the count after it? 4. Is it the opponent's last pearl, and if it is, what is the score? A doubtful answer to any of them is a reason to run. The side without the edge asks four as well. 1. Which pearl within reach is nearest to crossing? 2. Which of my pearls can take it with the fewest lost steps: one at home, the one

that will come off the count, or one that has to go back? 3. Are the captures level? If I am behind, is the pearl on the count? 4. What is the count after it, in turns, and how many captures bring the

target down to my score? A capture that leaves the count better than it was is made. Then both sides count again.

The Level Lock

Who has to open it, and what happens then

The level lock is the position in which two full lines face each other across the center. The counts are equal, and neither side can go forward without loss. This article makes three claims about it. The side that opens a level lock loses. The side that has to open it is the side that runs out of free steps first, and if both sides come straight up to the line that is the side that started the game ahead. So the lock is the natural aim of the side that starts behind, whichever color that is. Part One sets out the reasons. Part Two is about reaching the lock. Part Three is about what the side ahead can do to avoid it or to take it for itself. Part Four is about the play after the lock is opened, and is the fullest of the four. Part Five gives two games.

Part One
The Lock

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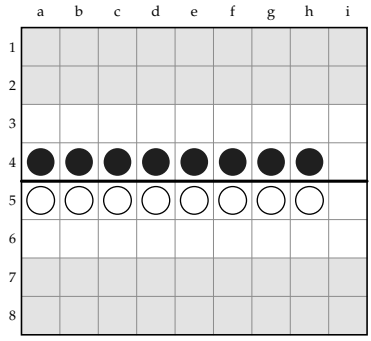
1. The Position

Diagram 1 shows the level lock. Each side has eight pearls in an unbroken line, Black on its fourth row and White on its fifth, and the two lines face each other across the center. Eight files are closed. The ninth, here the i-file, is open, because eight pearls cannot cover nine files. We call the end of a line that stands next to the open file its *open end*. The other end stands on the edge of the board.

Every pearl is five steps from crossing, so both counts are forty and each side needs ten turns.

Four facts about the position are used all through this article.



	●	○
+	0	0
x	0	0
Σ	40:10:0	40:10:0
∨	5	5

DIAGRAM 1. The level lock. Every pearl is five steps from crossing. Both counts are 40.

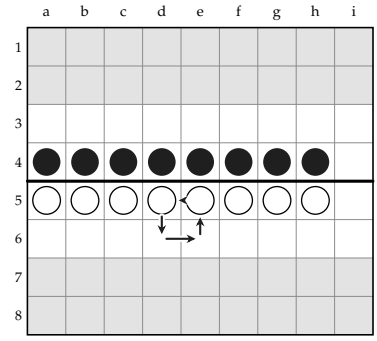


DIAGRAM 2. Marking time. White plays d5–d6, e5–d5, d6–e6, e6–e5. Two pearls have changed places and the line is as it was.

1. *There is one way forward.* Every square in front of a pearl is occupied. The only forward step on the board is the one into the open file, and only the pearl at the open end of each line can make it: h4 to i5 for Black, h5 to i4 for White.

2. *Nothing can be captured.* A pearl inside the line has two supporting pearls, and it takes six enemy pearls to capture it. Three at most can touch it from the front. The pearl at each end has one supporting pearl and can be captured by four, but only two enemy pearls touch it. The other two would have to stand beside the line or behind it, and there is nothing there.

3. *A gap changes that at once.* A pearl next to a gap in its own line has one supporting pearl, and an enemy pearl that steps into the gap is the fourth to touch it. Section 14 gives an example.

4. *The position is the same for both sides.* Turn the board round and change the colors, and nothing is different except who is to move.

2. The Count in a Lock

On the count the two sides are level, and equal counts go to the side that is to move. After equal turns it is Black's move, so on the count alone the lock would be Black's. *Arithmetic Margrites*, in this volume, discusses how the walk to the center ends in this position.

The count decides a race, and a lock is not a race. Neither side can run. If Black, to move, marks time for a turn, the position is the same and it is White's move, and by the same reasoning the lock now belongs to White. The count gives the lock to whichever side is to move, and the move changes hands every turn.

So the count leaves two questions open: what a side loses by opening the lock, and which side has to open it. The next four sections take them in turn. The side that opens the lock loses, and the side that has to open it is the side that has run out of other steps to play.

3. What the Side That Opens Loses

There is one way to open a level lock without losing a pearl on the spot. The pearl at the open end goes round the end of the enemy line: into the open file and on, four forward steps in a single turn, to the last row. We call it the *runner*. Nothing can catch it there, and no step has been lost. On the count the side that sent it is four steps ahead.

The runner has left its square empty, and that square is the way through for the other side. The other side's end pearl goes through it, also four forward steps, also to the last row. The counts are level again. Each line is a pearl shorter, each side has a runner behind the enemy line, and the side that opened the lock is to move.

It is to move in what is still a lock, seven against seven, and a worse one for it than the first. By fact 2 of Section 1, an end pearl is safe because nothing stands beside the line or behind it. Now something does. If the opener sends a second runner, its line is the shorter of the two, the pearl at the end of it has three enemy pearls in front of it, and the enemy runner comes back to be the fourth. If the opener waits instead, it is once more the side that has to give way. Part Four follows this play a turn at a time.

The side that opens a level lock loses.

Here "loses" means comes out behind. In every line we have examined, the side that opens the lock comes out behind, though seldom by much:

a pearl, or a single turn at the end of the game. The play that wins it is exact, and Part Four is given to it for that reason.

4. Waiting

A side that does not want to open the lock still has to play four steps every turn. There are three ways to find them.

A free step. A pearl that has not reached the line still has forward steps to make. These are the free steps of *Arithmetic Margrites*: steps toward the line that lose nothing on the count.

The long way round. A pearl with one step to go can take four to get there, by going sideways behind its own line first. The rule against stalling forbids a pearl to end its turn on the square where it began. It does not forbid a roundabout route to a new square.

Marking time. When every pearl is on the line, two pearls side by side can change places, as in Diagram 2. One steps back, the other steps across into its square, and the first comes round into the square the other has left. That is four steps, and the line stands exactly as it did. For a moment in the middle of the turn there is a gap in the line. It does no harm, because the opponent cannot move until the turn is over, and by then the gap is closed.

A side can mark time on every turn. If both sides do, the game stands still. The rules provide for this in the full hostile game: when both players have marked time, keeping their positions as they were, the player who marked time first must change its position. This article assumes that rule, or an agreement between the players to the same effect. Without it a level lock need never be opened and there is no more to be said about it.

We read the rule a little more widely than its words. A pearl can be stepped back from the line on one turn and forward again on the next, or walked to and fro behind it. That changes the position without getting anywhere. So we call a turn a *forward turn* only if it brings the side's count lower than it has yet been in the game, and we call every other turn a *waiting turn*. Marking time is a waiting turn. So is a step back, and so is the step forward that undoes it. A last free step played by the long way round is a forward turn, because the count falls by one for the first time. We apply the rule to waiting turns: when both sides have waited, the side that waited first must play a forward turn.

The first side to wait is the side that has to open the lock.

5. Who Waits First

A side with a free step to play has a forward turn and does not have to wait. So the question is which side runs out of free steps first.

A side's free steps are its count less forty, because the line is the place where every pearl is five steps from crossing. Suppose both sides come straight up to the line, playing four free steps a turn until they have none. The table gives the result for every arrangement. H and I, in which White puts one pearl forward or none, add steps to White's count and gain it nothing, as *Arithmetic Margrites* discusses, but they are included for completeness.

		Free steps		*First turn with*	
	Edge	*White*	*Black*	*no free step*	*Lock goes to*
A	White	16	18	White's 5th	Black
B	Black	17	19	Black's 6th	White
C	Black	18	20	Black's 6th	White
D	White	19	21	White's 6th	Black
E	White	20	22	White's 6th	Black
F	Black	21	23	Black's 7th	White
G	Black	22	24	Black's 7th	White
H	Black	23	24	Black's 7th	White
I	Black	24	24	Black's 7th	White

Take arrangement D. White has nineteen free steps and Black twenty-one. After four turns each side has played sixteen. On its fifth turn Black plays four more and has one left. White plays its last three, and finds its fourth step by the long way round. On its sixth turn Black plays its last free step, with three more by the long way round, and its line is complete. White's line has been complete for a turn. White has nothing to play and waits. Black waits in reply. White waited first, and now has to open the lock.

In every row the lock goes to the side that does not have the edge. That is not chance. Each side's free steps are its count less forty, so the walk to the line is the race itself, ten turns shorter. The side that would win the race is the side that reaches the line first, and the side that reaches the line first loses the lock.

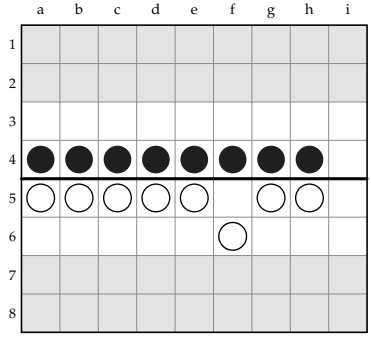
> *If both sides come straight up, the level lock goes to the side that started behind: Black in A, D and E, White in B, C, F, G, H and I.*

So in B, C, F, G, H and I a level lock does not keep Black's edge. It takes the edge from Black in those six arrangements, just as it takes it from White in the other three.

The setup rule gives Black one way to change this before the game begins. Black may put fewer pearls forward than the rule allows, and each pearl it holds back adds a free step. In B, a Black that puts three pearls forward instead of five has twenty-one free steps against White's seventeen. White then runs out first, on its sixth turn, and the lock goes to Black. Black then loses the edge on the count. Black's 61 against White's 57 is sixteen turns to fifteen, and White has the edge. In C one pearl held back does the same. In F and G Black cannot hold back enough to change the result, and in H and I it already has every pearl back. So in B and C Black can choose, before a step is played, between the edge on the count and the lock. In a gentlemen's game White may adjust its own arrangement in reply, and Black may answer in turn. Which is the better choice, and where that exchange ends, has not been worked out.

The rule has one important qualification: neither side has to come straight up.

6. A Step in Hand



	●	○
+	0	0
x	0	0
Σ	40:10:0	41:11:3
∨	5	6

DIAGRAM 3. White is one step short, inside its line, with the late pearl directly behind the gap. White has a step in hand, and Black will have to open.

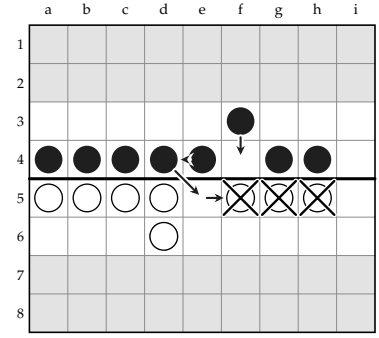


DIAGRAM 4. The late pearl stands to one side of the gap. Black steps in: d4–e5, f3–f4, e5–f5, e4–d4. The pearls on f5, g5 and h5 are captured.

A side need not play its free steps as fast as it can. It can keep a pearl one step short of the line and play that last step later. We say it has a *step in hand*. Three positions show what a side gains by a step in hand.

1. *Both lines complete.* This is the lock itself. The side to move has to wait first, and loses.
2. *One line complete, the other a step short*, as in Diagram 3. The side with the step in hand wins, whoever is to move. If it is to move, it plays the step, by the long way round, and the other side has to wait first. If the other side is to move, the other side has to wait first at once.
3. *Each side a step short.* The side to move loses. If it plays its step, it is in the second position on the wrong side. If it waits, the other side waits too, and then it has to play the step after all. *The lock goes to the side that plays the last free step.*

All this supposes that the gap can be left open safely for a turn or two, and that depends on where the late pearl stands. In Diagram 3 it stands directly behind the gap. It touches both of the pearls beside the gap, so each of them still has two supporting pearls and neither can be taken. An enemy pearl that steps into the gap is touched by three pearls at once, and it cannot go on, because the late pearl is in its way.

In Diagram 4 the late pearl stands to one side, on d6. It touches d5 but not f5. The pearl on f5 has one supporting pearl, and that is enough. Black steps into the gap with d4–e5, which is safe because e4 is beside it. Then f3–f4 completes Black's own line and puts three pearls in front of f5, with the pearl on e5 the fourth. It is captured. The pearl from e5 steps into its square, and g5 and h5 go the same way. Black's last step closes its own line again. Black has captured three pearls in one turn.

A gap at the end of the line is no better. The end square is the way round, and the other side goes through it at once. A pearl kept two rows back is worse still, and can simply be captured (Section 11). So a side can hold one step in hand safely, in one place: inside the line, with the late pearl directly behind the gap. We have not found a safe way to hold two.

Now go back to the table. On its last turn before the line the side ahead plays whatever free steps it has left, and in six of the nine arrangements that is exactly four. In the other three it is three, and one step of the turn is spare.

	Side ahead	*Free steps on its last turn to the line*	*Spare step*	*The lock*
A	White	4	none	Black's
B	Black	3	one	in dispute
C	Black	4	none	White's
D	White	3	one	in dispute
E	White	4	none	Black's
F	Black	3	one	in dispute
G	Black	4	none	White's
H	Black	4	none	White's
I	Black	4	none	White's

With a spare step the side ahead can play two of its three free steps, bring the third pearl round behind its gap, and keep the last step in hand. The side behind then plays its own last step, and finds that it was not the last. Part Three looks at this and at the replies to it.

Where the side ahead has no spare step, holding one back does not help it. In A, for instance, White would have to leave a pearl short on its fourth turn. Black then has two free steps to White's one, plays one and keeps one, and White is to move in the third position.

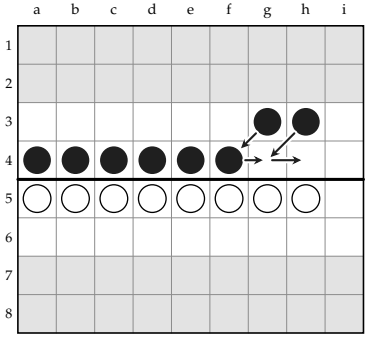
The three arrangements with a spare step are the three in which, on the count, the side with the edge has one step in hand. The six without are the six in which it has none. It is the same step, doing a second job.

> *In A, C, E, G, H and I the side behind takes the lock by coming straight up in good order. In B, D and F the side ahead has a step to spare, and there is a fight for the last step.*

Part Two

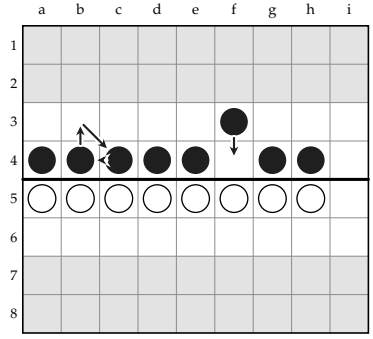
Getting There

7. The Tally



	●	○
+	0	0
x	0	0
Σ	42:11:2	40:10:0
∨	6	5

DIAGRAM 5. Two free steps and four to play. The pearl on f4 goes along the line to h4, and the two late pearls step up behind it.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	41:11:3	40:10:0	40:10:0	40:10:0
∨	6	5	5	5

DIAGRAM 6. One free step and four to play. Two pearls change places in three steps, b4–b3, c4–b4, b3–c4, and the late pearl plays f3–f4.

The side that wants the lock keeps one number for each side: the free steps it has left. A pearl two rows short of its line has two, a pearl one row short has one, and a pearl on the line has none. The side should take the tally before every turn from the third or fourth onward. It is used for two things.

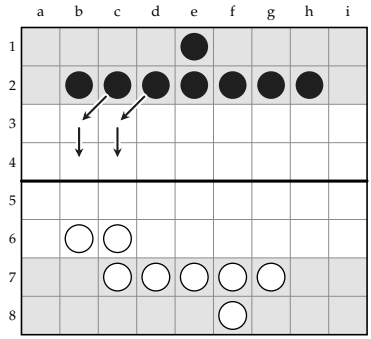
It shows which side will play the last free step. The rule is that of Section 6: more steps in hand wins, and with equal steps in hand the side to move loses. Near the end the numbers are small and the sums are easy.

It also shows how many of the four steps of a turn will be free and how many have to be found elsewhere. A turn with fewer than four free steps is filled out by the long way round, and the two diagrams show the usual ways. In Diagram 5 Black has two late pearls. A pearl already on the line

walks two squares along it, and the late pearls step up into the squares it has left. In Diagram 6 Black has one. Two pearls on the line change places in three steps, and the late pearl plays the fourth.

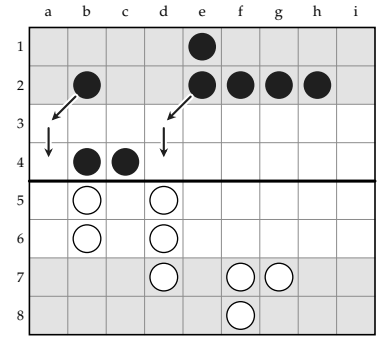
Steps played the long way round are lost steps on the count. In a lock that does not matter. Once the front is closed both sides lose four steps a turn, and the count that will matter is the one taken when the position opens. What matters about the turn is that it brought the count to a new low, so that it was a forward turn.

8. Order of Arrival



	●	○
+	0	0
x	0	0
Σ	57:15:3	55:14:1
∨	8	8

DIAGRAM 7. Black goes first where White is nearest, and arrives with two pearls together.



	●	○
+	0	0
x	0	0
Σ	53:14:3	51:13:1
∨	8	8

DIAGRAM 8. A pearl on each side of the pair. White's threat was b6–c5, d5–d4 and d6–d5.

A line cannot be built in one turn. For four or five turns part of it is on the fourth row and part is not, and the opponent is building a line of its own one row away. Four rules keep the part that has arrived from being taken, and keep the opponent off the squares that are still empty. The diagrams are from Game Two, where the whole walk can be followed.

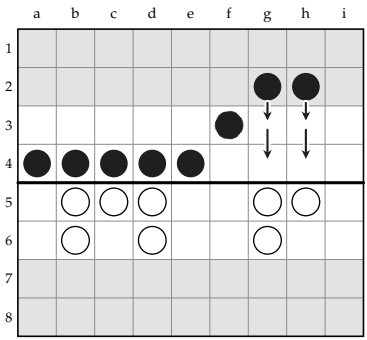
1. *Go first where the opponent is nearest.* The squares of your fourth row belong to whoever stands on them first with a supporting pearl. Where the opponent's pearls are far away there is no hurry. Where they are two rows off there is.

2. *Arrive in pairs.* A lone pearl on the line is captured by two enemy pearls. A pearl with a supporting pearl is captured only by four, and four seldom arrive in one turn. In Diagram 7 Black's first two pearls reach b4 and c4 together.

3. *Give the pair a third pearl before the fourth attacker comes.* After Diagram 7 White brought up four pearls, and with three more steps would have had four round c4. Black's reply in Diagram 8 puts a pearl on each side of the pair. A pearl with two supporting pearls needs six attackers, and six are not to be had against a pearl on the line.

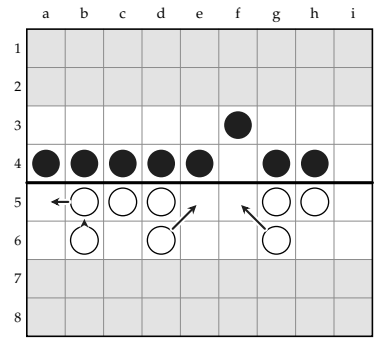
4. *Fill a square between two groups before the opponent can stand on it.* An enemy pearl on your fourth row with one of its own beside it is hard to remove, and your line stops where it stands. In Game Two Black's pearl from e1 makes three steps in one turn to reach e4 for this reason.

9. The Last Gap



	●	○
+	0	0
x	0	0
Σ	45:12:3	43:11:1
∨	7	6

DIAGRAM 9. Black finishes the end of the line and leaves the gap inside, at f4, with the late pearl on f3 behind it.



	●	○
+	0	0
x	0	0
Σ	41:11:3	43:11:1
∨	6	6

DIAGRAM 10. If White now completes its line, Black plays the last free step as in Diagram 6, and White has to wait first.

One turn before the line is complete it has a gap, and the opponent has the move. Which gaps are safe was discussed in Section 6. For the side building its line the rule is this.

Finish the two ends of the line first. Leave the last gap inside the line, with the late pearl directly behind it.

Diagram 9 is the turn in Game Two on which Black has to choose. It has five free steps left and four to play: one for the pearl on f3 and two each for the pearls on g2 and h2. It brings the two end pearls all the way and leaves f3 where it is. The pearl on f3 touches e4 and g4, so both have two supporting pearls. A white pearl stepping into f4 would be touched by all three.

If the gap were left at h4 instead, with the late pearl on h3, a white pearl could step into it from h5 with g5 beside it. It would be touched by only two black pearls, it would have the open file beside it to go on through, and Black's line would end a file short.

If White simply completes its line, as in Diagram 10, Black plays the last free step by the long way round, and that is the lock with White to wait first. In an arrangement with no spare step White has nothing better. In this one it has, and Part Three examines its choices.

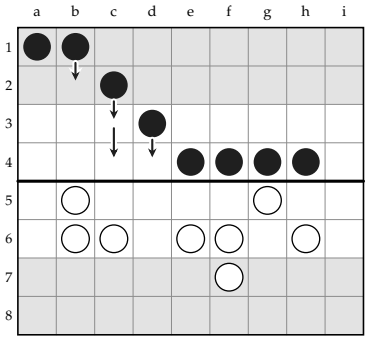
Part Three
What the Side Ahead Can Do

The side that started ahead wants a race and has no use for a lock. It has three ways to avoid one or to take it for itself. The first is open to it in every arrangement. The other two use the spare step, which it has only in B, D and F. It can also decline to build a line at all and fight in the open. That is a different game, and it is outside this article.

10. Attacking the Wing That Arrives Late

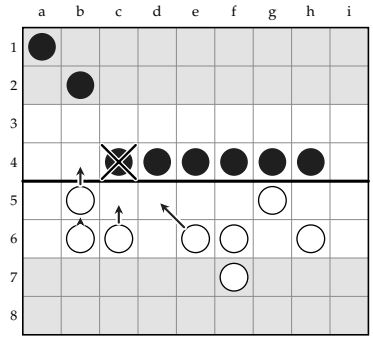
In Diagram 11 Black has built its line from the wrong side. On its first three turns it brought four pearls to the line on the files from e to h, where White was furthest away. Now it starts on the other wing, where White has been waiting with pearls on b5, b6, c6 and e6. The pearls on d4 and c4 arrive one after the other, and the pearl on c4 has only d4 beside it.

White needs four pearls round c4 and has them within a step: Diagram 12. White loses no steps in making the capture, since all four steps are



	●	○
+	0	0
x	0	0
Σ	49:13:3	47:12:1
∨	8	7

DIAGRAM 11. Black's fourth turn in another game in arrangement D. The pearl that reaches c4 has one supporting pearl and nothing behind it.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	1
Σ	45:12:3	47:12:1	40:10:0	36:9:0
∨	8	7	8	6

DIAGRAM 12. White's reply: b5–b4, b6–b5, c6–c5, e6–d5. Four pearls round c4, and it is captured.

forward, and it leaves a white pearl on b4, a row inside Black's half, with b5 and c5 behind it. Black is a pearl down and there will be no level lock on this wing.

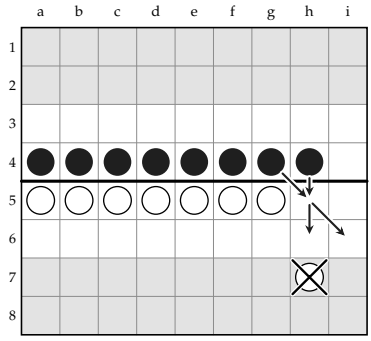
There is no reply at this point. The reply was the order of Section 8: go first where the opponent is nearest, and arrive there with two pearls and then a third. A side that must build a wing where the opponent's pearls are near should build it from the first turn.

11. Keeping the Last Step in Hand

In B, D and F the side ahead reaches its last turn before the line with three free steps and four to play. Its best use of the spare step is to keep one free step back. It plays two, brings the third pearl round until it stands directly behind its gap, and stops. If the side behind now plays its own last step, the side ahead plays the one it kept, and the side behind is the first with nothing to play.

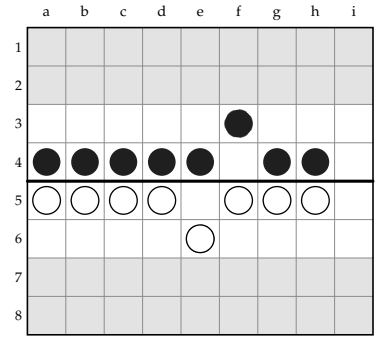
The side behind has three things to look for.

Is the late pearl two rows back? Then it is not a step in hand. It is a pearl on its own with a gap in front of it. In Diagram 13 two black pearls go through the gap and stand beside it, and it is captured. Black is a pearl up, has two pearls past the line, and needs eight turns to White's nine.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	1	0
Σ	40:10:0	42:11:2	31:8:1	35:9:1
∨	5	7	5	5

DIAGRAM 13. White has kept its last pearl two rows back. Black plays g4–h5, h5–i6, h4–h5, h5–h6, and the pearl on h7 is captured.



	●	○
+	0	0
x	0	0
Σ	41:11:3	41:11:3
∨	6	6

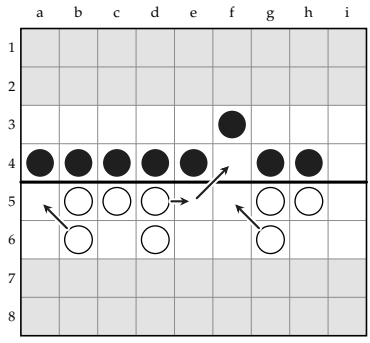
DIAGRAM 14. Each side has a step in hand behind a sound gap. Whichever side is to move is the worse off.

Is the late pearl to one side of its gap? Then the gap is not safe, and the reply is the step into it that Diagram 4 showed. It is an easy fault to fall into, because the late pearl often comes up from one side and it takes a sideways step to put it right.

Is the gap sound? Then the position is the third of Section 6, as in Diagram 14, with the side behind to move, and it is the worse off. It would like a step each with the opponent to move. For that it would have had to keep two steps back a turn earlier, and there is reason to doubt that two can be held (Section 6). Its best practical chance is to step into the gap anyway, with a pearl on each side in support. The gap is sound, so nothing is captured. The side behind gains a row, and the position is no longer a lock. The play that follows is open and difficult for both sides.

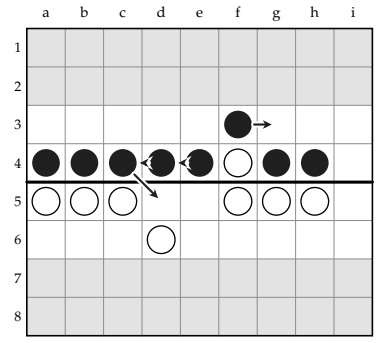
In these three arrangements, then, the lock is not something the side behind can count on. It can count on it only if the side ahead plays its last three free steps without thinking.

12. Stepping into the Last Gap



	●	○
+	0	0
x	0	0
Σ	41:11:3	43:11:1
∨	6	6

DIAGRAM 15. White uses its spare step to step in: d5–e5, e5–f4, with g6–f5 to stand behind it.



	●	○
+	0	0
x	0	0
Σ	41:11:3	40:10:0
∨	6	6

DIAGRAM 16. Black's reply: into the square White left, c4–d5, with the line closed up behind it, and the late pearl to g3.

The other use of the spare step is to put a pearl into the gap in the other side's line. Diagram 15 is White's fifth turn in Game Two. Black's gap is a sound one, so nothing can be captured. White's pearl on f4 has f5 and g5 beside it, and with two supporting pearls it cannot be captured either. It stays.

White has gained a row on the f-file. Its count is 40 and Black's 41, and the lock, if the position closed now, would not be level.

White has weakened its own line to gain the row. The pearl that stepped in came from d5, and the squares d5 and e5 are empty, with one white pearl behind them on d6. Black's reply in Diagram 16 has three parts.

1. *Step into the square the intruder left.* The pearl from c4 goes to d5. That is a row for a row, and the counts are level again at 40.

2. *Close the line behind it.* The pearls from d4 and e4 each move one file, so that a4 to d4 is a whole line again and the pearl on d5 has two supporting pearls.

3. *Give up the gap and guard the end.* The late pearl cannot have f4 now. It goes to g3, behind the end of the line. What it is for there is the subject of Section 19.

The game is open from here, and Game Two gives the rest of it. Black won it by a single turn. Of the three things the side ahead can do, this is the one that leaves the side behind most to play for.

Part Four
After the Lock

The lock is complete, and you are the side that played the last free step. The opponent has waited, you have waited, and the opponent has to open. This Part follows the play from there. The positions are from Game One, in which White is the side that has to open.

13. Waiting Well

While the lock stands, the side that holds it should change nothing. It marks time, as in Diagram 2. Any two pearls side by side will do.

Three things are to be avoided. Do not answer the opponent's marking time with a runner. The opponent played a waiting turn, and if you wait in reply the opponent has waited first. Do not step a pearl back from the line to have something to do next turn. The step forward again is not a forward turn, so nothing is gained, and unless the pearl stands directly behind its square something may be lost. And if you still have a step in hand, do not be in a hurry to play it. It is the only forward turn you have that loses nothing.

14. Why the Line Cannot Be Changed

The side that has to open would like to do it without sending a pearl away. It might move the whole end of its line one file toward the open side, as in Diagram 17, and hold a pearl in the open file. That leaves a gap inside the line, and Diagram 18 shows what the other side does with a gap inside the line.

The pearl on d5 stands beside the gap. It has one supporting pearl, c5, and three black pearls in front of it. Black's pearl from f4 steps into the

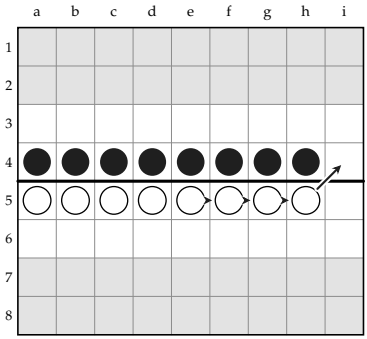


DIAGRAM 17. White tries to change its line without sending a runner: h5–i4, g5–h5, f5–g5, e5–f5. There is a gap at e5.

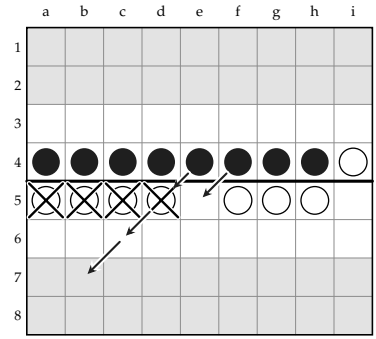


DIAGRAM 18. Black plays f4–e5, e4–d5, d5–c6, c6–b7. The pearls on d5, c5, b5 and a5 are captured, one with each step.

gap, which it can do because d4 and e4 are beside it, and it is the fourth. The pearl on d5 is captured. Now c5 is the end of a line: one supporting pearl, three black pearls in front. The pearl from e4 steps into d5 and is the fourth again. Two more steps take b5 and a5 in the same way. Four steps, four captures, and every step was forward.

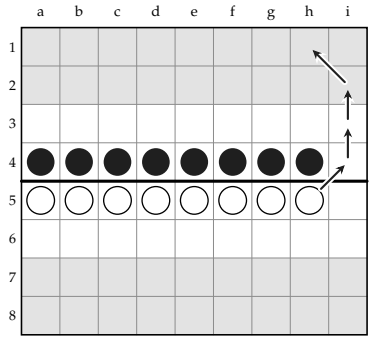
Other changes do no better. A pearl stepped back from the open end leaves the end square to the other side. A pearl stepped straight back from inside the line loses nothing at once, but that is a waiting turn, and so is the turn that brings it forward again. The only forward turn that does not lose a pearl is the runner's.

15. The Runner and the Reply

The runner in Diagram 19 goes from the open end of the line to the last row in one turn. It has to make all four steps. A runner that stops beside the end of the enemy line, on i4 or i3, has one black pearl touching it, another a step away, and no supporting pearl.

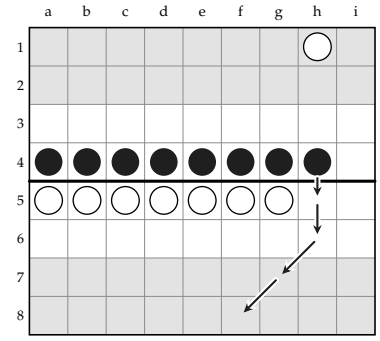
The reply is in Diagram 20, and it should be played at once. The runner's square, h5, is empty. Black's end pearl goes straight through it and on to the last row. The counts are level at 36, and each side has a pearl one step from scoring.

There are two other things Black might think of, and neither is good. It cannot catch the runner, which is on the last row with nothing near it. Section 19 deals with the case in which something is near it. And it should



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	40:10:0	40:10:0	40:10:0	36:9:0
∨	5	5	5	5

DIAGRAM 19. White opens. The runner goes round the end: h5–i4, i4–i3, i3–i2, i2–h1. White 36, Black 40.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	40:10:0	36:9:0	36:9:0	36:9:0
∨	5	5	5	5

DIAGRAM 20. Black replies through the square the runner left: h4–h5, h5–h6, h6–g7, g7–f8. 36 against 36.

not wait. If Black marks time, the runner scores, White's line of seven still stands, and White is five steps ahead on the count, having lost nothing.

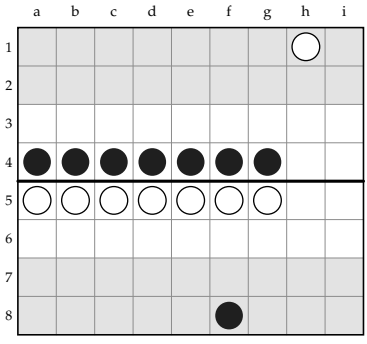
16. The Lock Again, One File Shorter

Diagram 21 is the position after the two runners. It is the level lock again. The lines are seven pearls long, both counts are 36, and the position is the same from both sides, as before. And White is to move, as before.

White can wait. If it does, Black must wait too. This is the point at which Black most often loses the game, because Black's second pearl has the same route in front of it that the first one took, and it is tempting to send it. Diagram 22 shows the result. Black's line would then be six pearls long and White's seven. The pearl at the end of Black's line, on f4, would have three white pearls in front of it, e5, f5 and g5. White's pearl from g5 steps down beside it and the runner comes back from h1 to g3. Four pearls round f4, and it is captured.

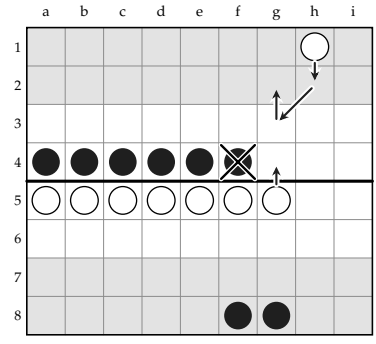
> *After the first exchange of runners, do not send a second. Wait. The side that opened the lock is still the side that has to move.*

The reason is the same at every stage. An end pearl is safe while only two enemy pearls stand in front of it. When a line becomes shorter than



	●	○
+	0	0
x	0	0
Σ	36:9:0	36:9:0
∨	5	5

DIAGRAM 21. After a runner each. Seven against seven, a runner behind each line, and White to move again.



	●	○
+	0	0
x	0	0
Σ	32:8:0	36:9:0
∨	5	5

DIAGRAM 22. If White had waited and Black had sent the second runner, White would capture in the same way: g5–g4, h1–h2, h2–g3, and f4 is taken.

the line facing it, its end pearl has three in front, and a runner that is already behind the line makes four.

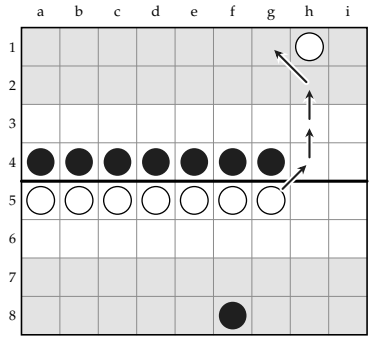
17. The Second Runner and the Capture from Behind

White has waited once and cannot wait again, or it does not choose to. Its second runner, in Diagram 23, goes through the square that Black's runner left. White's count is 32.

White's line is now six pearls long and Black's is seven. The pearl on f5 is the end of White's line. It has one supporting pearl, on e5. In front of it stand the black pearls on e4, f4 and g4.

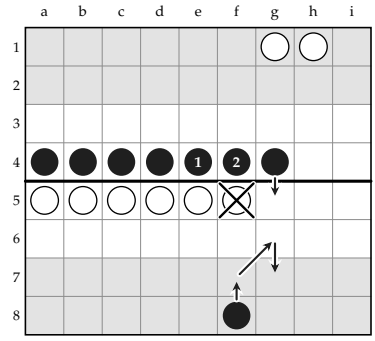
Diagram 24 is the capture. Black's first step is g4–g5, a forward step, which puts the pearl from g4 beside f5. The pearls numbered 1 and 2 are still in front of it. Then the runner turns back. Two steps, f8–f7 and f7–g6, bring it to the square behind f5, and it is the fourth. The pearl on f5 is captured on the third step of the turn. With the fourth step the runner starts forward again.

Black has lost four steps, because two of the runner's steps went backward and each backward step is two steps lost. In return Black has made the first capture of the game. The next section shows why Black gains more by that capture than the four steps it lost.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	36:9:0	36:9:0	36:9:0	32:8:0
∨	5	5	5	5

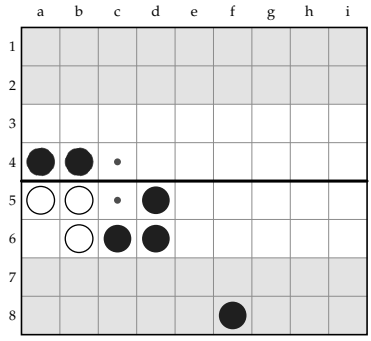
DIAGRAM 23. White sends a second runner: g5–h4, h4–h3, h3–h2, h2–g1. Its line now ends at f5.



	●	○
+	0	0
x	0	0
Σ	36:9:0	32:8:0
∨	5	5

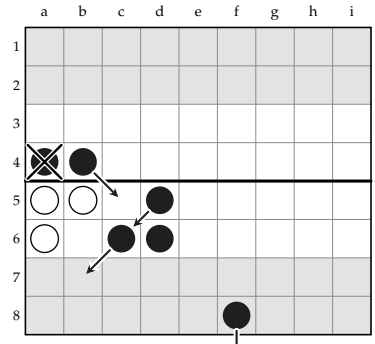
DIAGRAM 24. Black plays g4–g5 and brings the runner back, f8–f7, f7–g6. Four pearls touch f5, and it is captured. The runner goes forward again.

18. The First Capture and the Far Wing



	●	○
+	1	3
x	2	1
Σ	16:4:0	16:4:0
∨	5	6

DIAGRAM 25. Game One, before White's eleventh turn. White's last three pearls have no forward step. The squares c4 and c5 are shut to them.



	●	○
+	1	3
x	2	1
Σ	16:4:0	16:4:0
∨	5	6

DIAGRAM 26. Black lets the wall go. With b4–c5 the pearl on a4 is left alone and captured. Black did not need it.

As *Arithmetic Margrites* discusses, the side ahead in captures does not need all of its pearls. The target is the lower of the two maximums. Each capture you are ahead by lowers the target for you and leaves you one pearl more than you need. The side that is behind in captures needs every pearl it has.

After the lock has been opened, this has a particular meaning, because of where the pearls are. The runners all left from the open end. The pearls at the other end of each line, on the a, b and c files, have not moved. They are still face to face. For the side that is behind in captures, those are pearls that have to get home, and they are standing behind an enemy line. For the side that is ahead, its own pearls on those files are the ones it does not need. They can stay where they are for as long as they are useful, and what they are useful for is standing in the way.

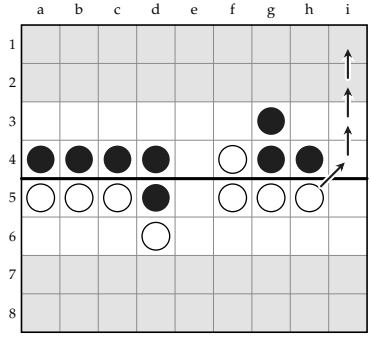
Diagram 25 is Game One nine turns later. White has scored three and has three pearls left, on a5, b5 and b6. It needs all three. Black's pearls on a4 and b4, ringed, have not moved since the lock was formed. Black has brought three more pearls round behind, to c6, d5 and d6. White is to move and has no forward step on the board. The square c4 is empty, but a pearl stepping there would stand beside b4 and d5 with no supporting pearl. The square c5 has four black pearls round it. White has to play four steps that go nowhere. Before this turn both sides needed four turns and White was to move, so White was ahead. After it they still need four each, and Black is to move.

> *The side that makes the first capture after the lock has a wall it does not have to move. The other side has pearls behind that wall that must get home.*

A wall has to be let go at the right time. Its pearls are five steps from home, and while the target still counts one of them, the side that owns the wall cannot finish. In Diagram 26 Black's other pearls are within three turns of home and it is Black's move. It steps away from a4, and the pearl is captured. Black had six pearls on the board and needed five crossings, so one pearl was spare, and the spare one was on a4. With the wall gone, every step Black plays is a step by a pearl that counts. Game Two has the same moment, with two pearls given up in one turn.

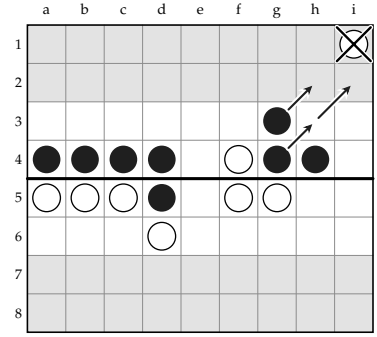
The time to let the wall go is found by counting twice, once with the wall and once without. With it, the side counts its nearest pearls up to the target and the wall holds the enemy for another turn. Without it, the target may fall and the enemy pearls are free. When the count without the wall is a turn better than the opponent's, the side should let the wall go.

19. A Pearl Behind the End



	●	○
+	0	0
x	0	0
Σ	40:10:0	40:10:0
∨	6	6

DIAGRAM 27. Game Two. Black has three pearls together by the open file, one of them behind the line. White sends a runner all the same.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	1	0
Σ	40:10:0	36:9:0	36:9:0	35:9:1
∨	6	6	7	6

DIAGRAM 28. The runner is caught on the last row: g4–h3, h3–i2 and g3–h2 put two pearls beside it.

Everything in this Part so far has supposed that a runner which reaches the last row is safe. It is safe when the other side has all eight pearls on its line. It is not safe when one of them stands behind the open end.

In Diagram 27 Black has seven pearls in the front and one, on g3, behind the end of its line. White's runner goes round the end in the usual way and reaches i1. In Diagram 28 the two black pearls nearest the open file go back after it. The runner took four steps to get there. They need three to stand on i2 and h2, and a runner on the last row with two enemy pearls beside it and no supporting pearl is captured.

The pearl taken was one step from crossing. Black's steps were backward steps, and its pearls on that wing are further from home. Black is a capture ahead, and the wall of the last section still stands.

For the side that has to open, the lesson is to look behind the end of the line before sending a runner. For the side that waits, it is that the eighth pearl does not have to be on the line. A line of seven with the eighth pearl behind its open end is a lock as well, and in the lines we have examined the side to move in it is the worse off, as in the lock of eight.

20. The Play in Order

For the side that holds the lock, the whole of this Part comes to six instructions.

1. While the lock stands, mark time. Change nothing in the line.
2. When the runner goes round the end, send your own end pearl through the square it left, on the same turn.
3. Then wait again. Do not send a second runner.
4. When the opponent's second runner goes, take the end of its line: three pearls in front and your runner behind.
5. Leave the far end of your line where it is. It is holding the pearls the opponent needs.
6. Let it go when the count without it is a turn better than the opponent's.

Part Five
Two Games

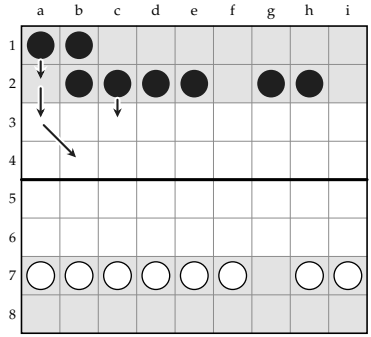
Each game is given in full. A diagram shows the position before the turn, with arrows for the steps played in it and a cross on any pearl captured in it, and the table beneath it gives the figures before and after the turn. The last diagram of a game shows where it ends. After each game we set out why the winner won and where the loser went wrong.

21. Game One: The Lock Taken and Won

Arrangement A. White has all eight pearls on its front row and Black has six. White's count is 56, fourteen turns. Black's is 58, fifteen turns. White has the edge with nothing in hand. White has sixteen free steps to the line and Black eighteen, so White will be there first, and it will have no step to spare on the way. Black's plan is the lock.

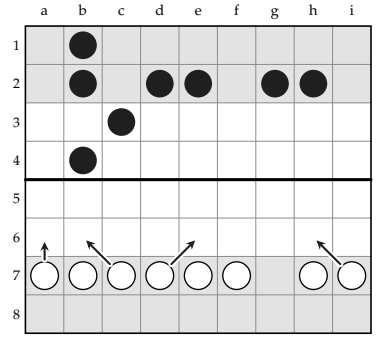
Black begins with the pearl that has furthest to go. The back-row pearl on a1 plays all three of its free steps at once and stands on the line at b4.

White starts forward on both wings. Black has fourteen free steps left and White twelve.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	58:15:2	56:14:0	54:14:2	56:14:0
∨	8	7	8	7

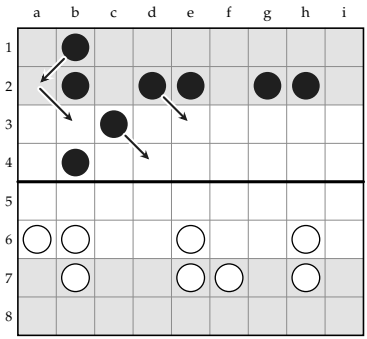
DIAGRAM 29. Black's first turn: a1–a2, a2–a3, a3–b4, c2–c3.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	54:14:2	56:14:0	54:14:2	52:13:0
∨	8	7	8	7

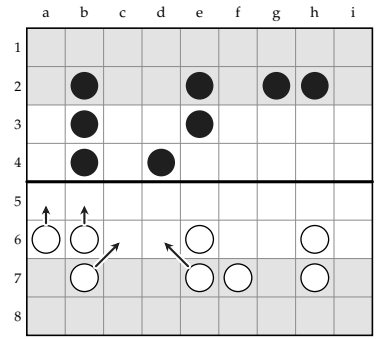
DIAGRAM 30. White's first turn: d7–e6, i7–h6, a7–a6, c7–b6.

The second back-row pearl comes up to b3, behind the first, so that the pearl on b4 is not alone when White arrives. White arrives on its next turn: its pearls on a5 and b5 are on their line and both touch b4. Two enemy pearls take a pearl that stands alone. They do nothing to a pearl with a supporting pearl. Black has ten free steps left, White eight.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	54:14:2	52:13:0	50:13:2	52:13:0
∨	8	7	7	7

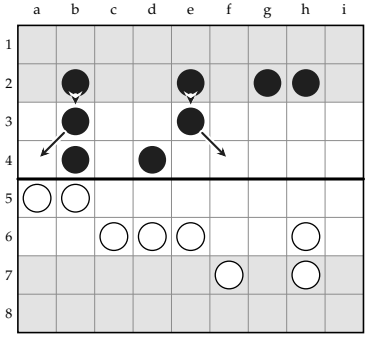
DIAGRAM 31. Black's second turn: b1–a2, a2–b3, d2–e3, c3–d4.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	50:13:2	52:13:0	50:13:2	48:12:0
∨	7	7	7	7

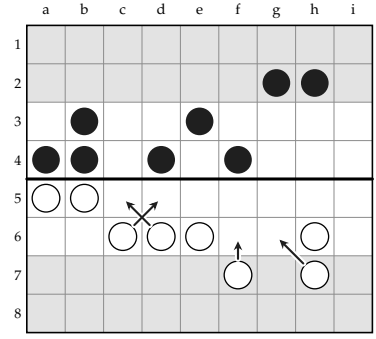
DIAGRAM 32. White's second turn: e7–d6, b7–c6, a6–a5, b6–b5.

Black has four pearls on the line, on a4, b4, d4 and f4, two a row behind them, and two still on their second row. White has four on its line, from a5 to d5, and four a row short. Black six free steps, White four.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	50:13:2	48:12:0	46:12:2	48:12:0
∨	7	7	7	7

DIAGRAM 33. Black's third turn: b3–a4, e3–f4, e2–e3, b2–b3.

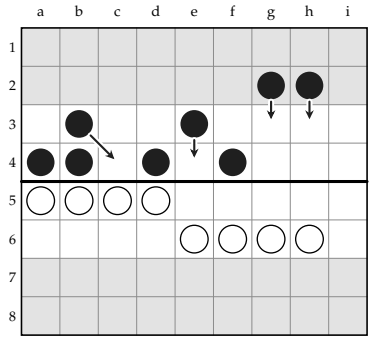


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	46:12:2	48:12:0	46:12:2	44:11:0
∨	7	7	7	6

DIAGRAM 34. White's third turn: f7–f6, h7–g6, d6–c5, c6–d5.

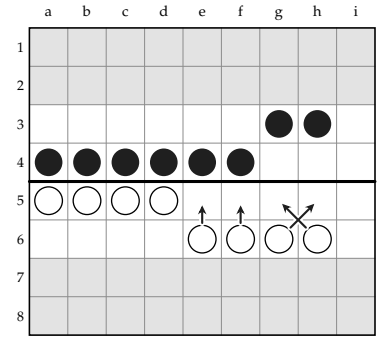
Black fills c4 and e4 and brings its last two pearls to the row behind the line. White has exactly four free steps and plays them. Its line is complete from a5 to h5, and it has had no step to spare: in arrangement A its sixteen

free steps make four full turns. Black has two free steps left, for the pearls on g3 and h3.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	46:12:2	44:11:0	42:11:2	44:11:0
∨	7	6	6	6

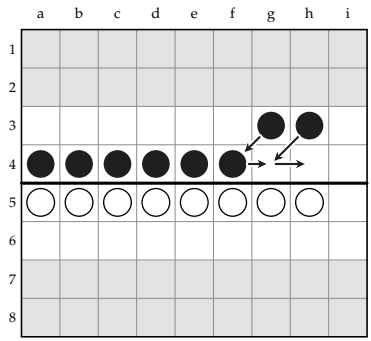
DIAGRAM 35. Black's fourth turn: e3–e4, h2–h3, b3–c4, g2–g3.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	42:11:2	44:11:0	42:11:2	40:10:0
∨	6	6	6	5

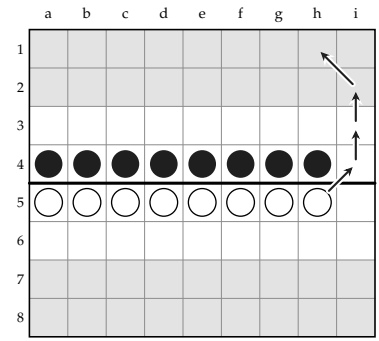
DIAGRAM 36. White's fourth turn: e6–e5, f6–f5, h6–g5, g6–h5.

Black has two free steps and four to play. The pearl on f4 walks two squares along the line to h4, and the pearls from h3 and g3 step up into g4 and f4. That is the long way round, and it is a forward turn: Black's count reaches 40 for the first time.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	42:11:2	40:10:0	40:10:0	40:10:0
∨	6	5	5	5

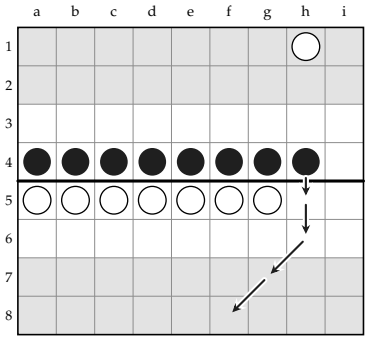
DIAGRAM 37. Black's fifth turn: f4–g4, g4–h4, h3–g4, g3–f4.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	40:10:0	40:10:0	40:10:0	36:9:0
∨	5	5	5	5

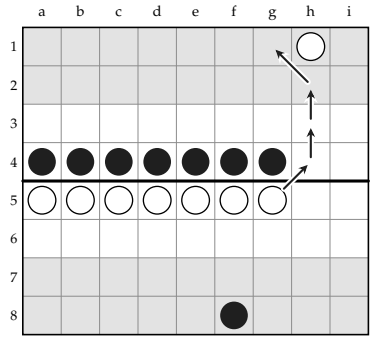
DIAGRAM 38. White's fifth turn: h5–i4, i4–i3, i3–i2, i2–h1.

This is the level lock, and White is to move with no forward turn except the runner. It could mark time once. Black would mark time in reply, and White, having waited first, would then have to open in any case. White opens at once. The runner goes round the end to the last row. White 36, Black 40. Black answers through the square the runner left. Its end pearl goes through h5 to the last row, and the counts are level at 36. The position is the lock again, seven against seven, and White is to move again.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	40:10:0	36:9:0	36:9:0	36:9:0
∨	5	5	5	5

DIAGRAM 39. Black's sixth turn: h4–h5, h5–h6, h6–g7, g7–f8.

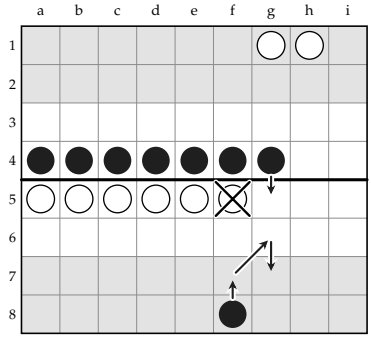


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	36:9:0	36:9:0	36:9:0	32:8:0
∨	5	5	5	5

DIAGRAM 40. White's sixth turn: g5–h4, h4–h3, h3–h2, h2–g1.

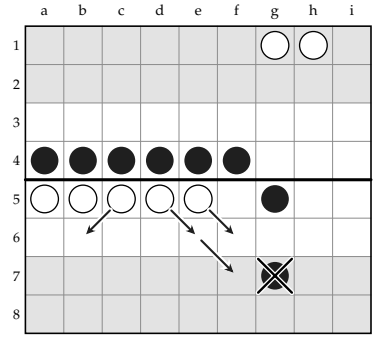
White sends a second runner, through the square Black's pearl has just left. Its count is 32, but its line is now one pearl shorter than Black's.

White's line ends at f5. That pearl has one supporting pearl, and the black pearls on e4, f4 and g4 stand in front of it. The pearl on g4 steps forward to g5, still touching it. The runner comes back two rows, from f8 to g6, and is the fourth. The pearl on f5 is captured, and the runner turns forward again. Black is a capture ahead. The target is seven, and Black has eight pearls.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	1	0
Σ	36:9:0	32:8:0	31:8:1	27:7:1
∨	5	5	5	5

DIAGRAM 41. Black's seventh turn: g4–g5, f8–f7, f7–g6, g6–g7. The pearl on f5 is captured.

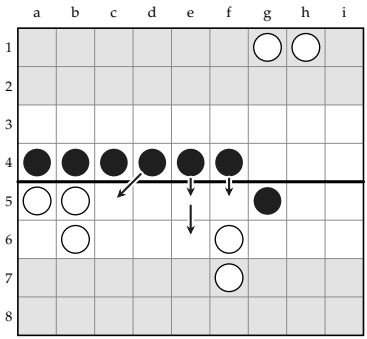


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	1	0	1	1
Σ	31:8:1	27:7:1	34:9:2	31:8:1
∨	5	5	5	7

DIAGRAM 42. White's seventh turn: c5–b6, d5–e6, e5–f6, e6–f7. The pearl on g7 is captured.

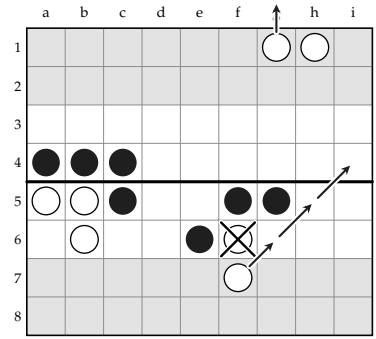
White hits back at the runner. Three pearls leave its line and go backward, and the two that reach f6 and f7 take the black pearl on g7, which has no supporting pearl. The captures are level at one each and the target is seven for both. White made four backward steps to do it, and it has no line any more. Black needs 34 steps and White 31.

Black goes forward into the squares White has left. Three pearls cross the center, and the pearls on e6, f5 and g5 now stand round the white pearl on f6.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	1	1	1	1
Σ	34:9:2	31:8:1	30:8:2	31:8:1
∨	5	7	5	7

DIAGRAM 43. Black's eighth turn: d4–c5, e4–e5, f4–f5, e5–e6.

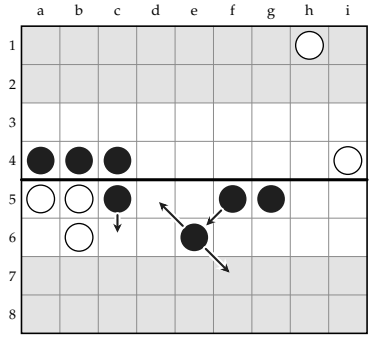


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	1
x	1	1	2	1
Σ	30:8:2	31:8:1	25:7:3	21:6:3
∨	5	7	5	6

DIAGRAM 44. White's eighth turn: f7–g6, g6–h5, h5–i4, g1–off. The pearl on f6 is captured.

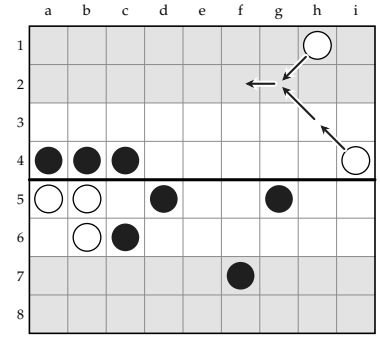
White cannot save both of its back pearls. It runs with the one on f7 and lets the other go: when the pearl from f7 takes its second step, f6 has no supporting pearl and three black pearls beside it, and it is captured. White has given up one of its two slowest pearls, six steps from home, and it scores with the runner on g1. Black has two captures to one, and the target is six. White needs 21 steps for five more crossings, six turns. Black needs 25 for six, seven turns.

On the count White is ahead, six turns against seven. Sixteen of its 21 steps belong to the three pearls on a5, b5 and b6, and those three stand behind the black pearls on a4, b4 and c4, which have not moved since the lock. Black brings two more pearls round them, to c6 and d5. The pearl from f5 uses two steps getting to d5, one of them backward, so Black's count falls by two. Black needs 23.



	BEFORE		AFTER	
	●	○	●	○
+	0	1	0	1
x	2	1	2	1
Σ	25:7:3	21:6:3	23:6:1	21:6:3
∨	5	6	5	6

DIAGRAM 45. Black's ninth turn: e6–f7, f5–e6, e6–d5, c5–c6.

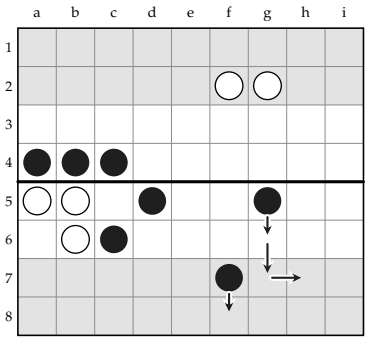


	BEFORE		AFTER	
	●	○	●	○
+	0	1	0	1
x	2	1	2	1
Σ	23:6:1	21:6:3	23:6:1	20:5:0
∨	5	6	5	6

DIAGRAM 46. White's ninth turn: i4–h3, h3–g2, g2–f2, h1–g2.

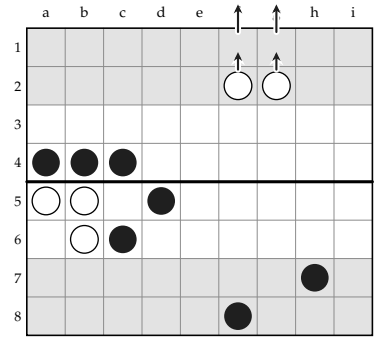
White brings its two free pearls nearer home, but slowly: one sideways step and one backward step among the four. It has nothing useful to do with the other three pearls, four steps must be played every turn, and these are the ones that lose the fewest steps. White needs 20.

Black puts two pearls on its last two rows and needs 20. White scores twice and has three. It is left with the three pearls on the left, sixteen steps from home between them, which is four turns. Black needs five turns and it is Black's move. White is still a turn ahead on the count.



	BEFORE		AFTER	
	●	○	●	○
+	0	1	0	1
x	2	1	2	1
Σ	23:6:1	20:5:0	20:5:0	20:5:0
∨	5	6	5	6

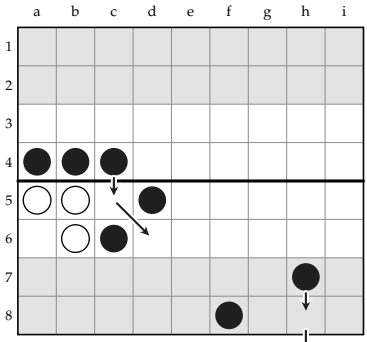
DIAGRAM 47. Black's tenth turn: f7–f8, g5–g6, g6–g7, g7–h7.



	BEFORE		AFTER	
	●	○	●	○
+	0	1	0	3
x	2	1	2	1
Σ	20:5:0	20:5:0	20:5:0	16:4:0
∨	5	6	5	6

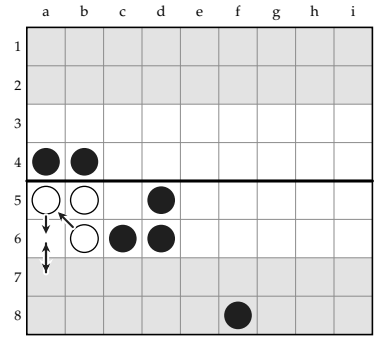
DIAGRAM 48. White's tenth turn: f2–f1, f1–off, g2–g1, g1–off.

Black scores its first pearl and moves the pearl from c4 to d6. That takes a pearl out of the wall, and it does not matter, because the squares it guarded are still shut. A white pearl stepping to c4 would stand beside b4 and d5 with no supporting pearl, and c5 has four black pearls round it.



	BEFORE		AFTER	
	●	○	●	○
+	0	3	1	3
x	2	1	2	1
Σ	20:5:0	16:4:0	16:4:0	16:4:0
∨	5	6	5	6

DIAGRAM 49. Black's eleventh turn: c4–c5, c5–d6, h7–h8, h8–off.



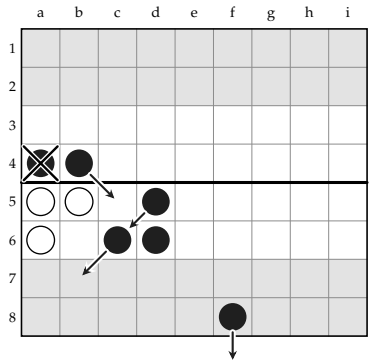
	BEFORE		AFTER	
	●	○	●	○
+	1	3	1	3
x	2	1	2	1
Σ	16:4:0	16:4:0	16:4:0	16:4:0
∨	5	6	5	6

DIAGRAM 50. White's eleventh turn: a5–a6, a6–a7, a7–a6, b6–a5.

White has no forward step on the board. Its three pearls shuffle: the one on a5 goes back and round to a6, and the one on b6 comes down into its

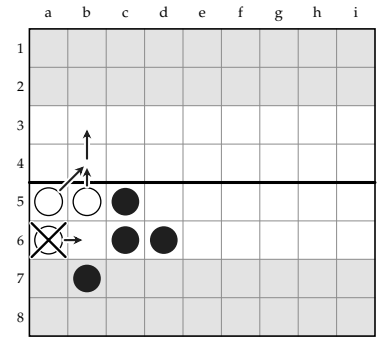
place. Four steps lost. Both sides now need sixteen steps, four turns, and it is Black's move. The turn White was ahead by has gone.

Black lets the wall go. The pearl on b4 steps forward, and the pearl on a4, left alone beside two white pearls, is captured. Black gave it up on purpose. It had six pearls on the board and needed five crossings, and a4 was the one it could spare. All four of Black's steps are forward: it scores a second pearl and needs twelve more steps, three turns. White still needs sixteen, four turns.



	BEFORE		AFTER	
	●	○	●	○
+	1	3	2	3
x	2	1	2	2
Σ	16:4:0	16:4:0	12:3:0	16:4:0
∨	5	6	4	6

DIAGRAM 51. Black's twelfth turn: b4–c5, c6–b7, f8–off, d5–c6. The pearl on a4 is captured.

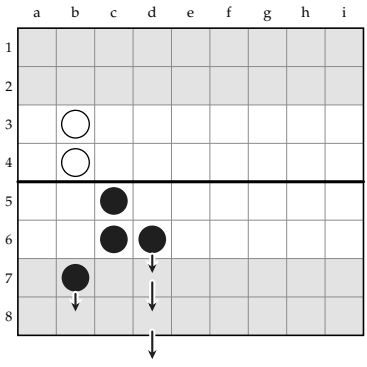


	BEFORE		AFTER	
	●	○	●	○
+	2	3	2	3
x	2	2	3	2
Σ	12:3:0	16:4:0	8:2:0	7:2:1
∨	4	6	3	4

DIAGRAM 52. White's twelfth turn: a6–b6, b5–b4, b4–b3, a5–b4. The pearl on b6 is captured.

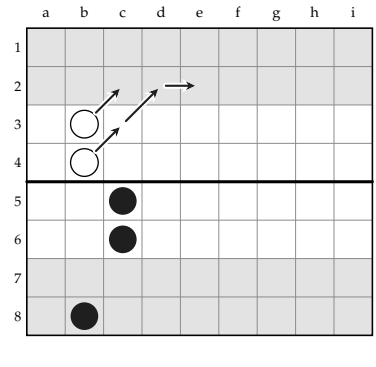
White does the same thing and has to. Its pearl from a6 steps sideways to b6, next to three black pearls, and the other two run. When they have gone the pearl on b6 has no supporting pearl and is captured. It was six steps from home. The target falls to five. White needs two more crossings, seven steps, two turns. Black needs three more, from the pearls on b7, c6 and d6: eight steps, two turns. And it is Black's move.

Black scores a third pearl and puts another on the last row. It needs four more steps, exactly one turn. White comes down as fast as it can. It also needs four steps and one turn, but its turn comes second.



	BEFORE		AFTER	
	●	○	●	○
+	2	3	3	3
x	3	2	3	2
Σ	8:2:0	7:2:1	4:1:0	7:2:1
∨	3	4	3	4

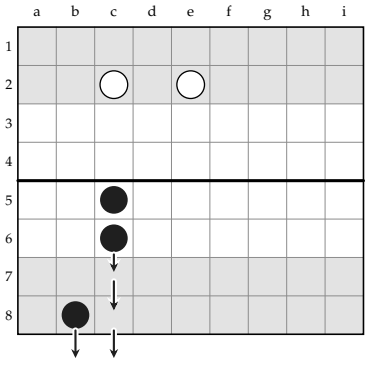
DIAGRAM 53. Black's thirteenth turn: d6–d7, d7–d8, d8–off, b7–b8.



	BEFORE		AFTER	
	●	○	●	○
+	3	3	3	3
x	3	2	3	2
Σ	4:1:0	7:2:1	4:1:0	4:1:0
∨	3	4	3	2

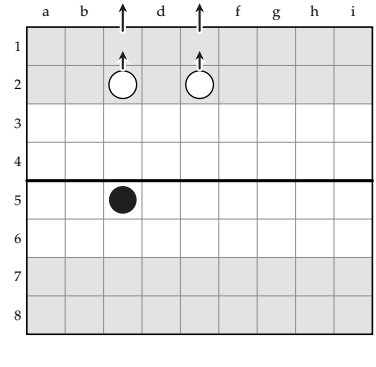
DIAGRAM 54. White's thirteenth turn: b3–c2, b4–c3, c3–d2, d2–e2.

Black scores twice and has five, which is the target. The game is decided on this turn although it is not over. Black's last pearl stays on c5 and never has to move.



	BEFORE		AFTER	
	●	○	●	○
+	3	3	5	3
x	3	2	3	2
Σ	4:1:0	4:1:0	0:0:0	4:1:0
∨	3	2	–	2

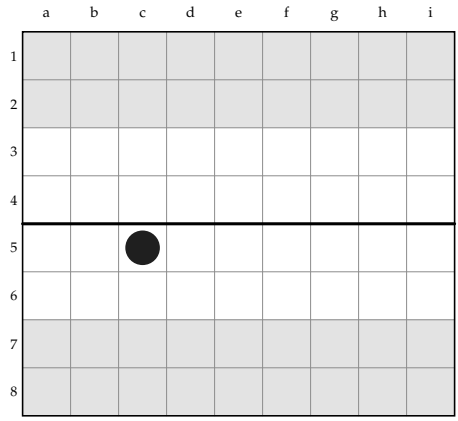
DIAGRAM 55. Black's fourteenth turn: b8–off, c6–c7, c7–c8, c8–off.



	BEFORE		AFTER	
	●	○	●	○
+	5	3	5	5
x	3	2	3	2
Σ	0:0:0	4:1:0	0:0:0	0:0:0
∨	–	2	–	–

DIAGRAM 56. White's fourteenth turn: c2–c1, c1–off, e2–e1, e1–off.

White crosses its last two pearls. That makes five all, White has no pearls left, and the game ends. Black has made three captures and White two.



	●	○
+	5	5
x	3	2
Σ	0:0:0	0:0:0
∨	–	–

DIAGRAM 57. The end. Five all, and Black wins on captures, three to two.

Why Black won. Black won this game by doing four things in order, each of which this article sets out as a rule.

First, Black took the lock. In arrangement A White starts ahead with no spare step, so the lock goes to Black if Black comes up in good order (Section 6). Black did. It brought its back-row pearls up first, because they had furthest to go. It never left a pearl alone on the line when a white pearl could reach it: on its second turn the pearl on b3 stood behind b4 before White arrived on a5 and b5. And it played its last free steps the long way round, so that its fifth turn was a forward turn and White was left to move with nothing but a runner.

Second, Black answered the runner at once, through the square the runner left (Section 15). The counts were level again, and White was again the side to move.

Third, when White sent a second runner, Black took the end of White's line with three pearls in front of it and its own runner behind (Section 17). That was the first capture of the game. From then on Black needed one pearl fewer than it had, and White needed every pearl it had.

Fourth, Black used the far end of its line as a wall and let it go at the right time (Section 18). White's three pearls on a5, b5 and b6 had sixteen of White's steps between them and could not get past the black pearls on a4, b4 and c4. On its eleventh turn White had no forward step on the board and lost four steps, which was the whole of the turn it had been ahead by. On its own twelfth turn Black gave up the pearl on a4, which it

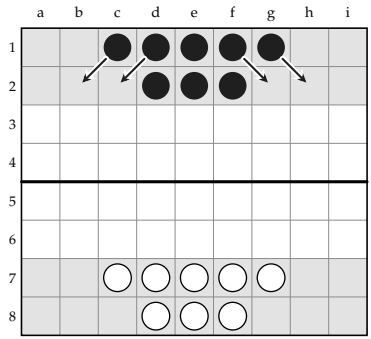
did not need, and every step it played after that was a step by a pearl that counted.

Where White went wrong. White did not lose this game on one turn. Once Black had the lock, White had to open it, and every way of opening loses something (Section 3). The turns on which White lost most were these. On its sixth turn it sent a second runner, which made its line one pearl shorter than Black's and left the end pearl on f5 with three black pearls in front of it. On its seventh turn it took Black's runner back, but it did so with three backward steps from its line. Black walked into the squares White had left, and on its next turn White could not save both of its back pearls and lost one of them. After that White was behind in captures, and the three pearls it needed most were the three standing behind Black's wall. The game ended five all, and Black won on captures, three to two.

22. Game Two: The Step into the Gap

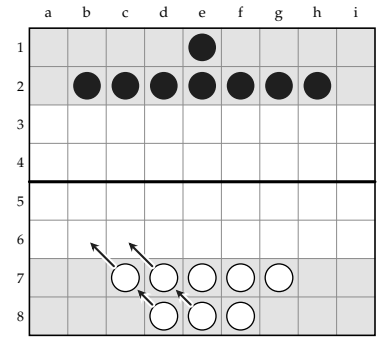
Arrangement D. White has five pearls forward, on c7 to g7, and three behind them. Black has three forward, on d2, e2 and f2, and five behind. White's count is 59 and Black's 61: fifteen turns against sixteen, White's edge with one step in hand. White has nineteen free steps to the line and Black twenty-one. This is one of the three arrangements in which the side ahead has a step to spare on its last turn before the line, and the game shows White using it.

Black spreads its back row. All four steps are diagonal and forward, so they are free steps: the pearls change file without losing a step. White moves its pearls toward one side. Four of them now stand on the b, c and d files.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	61:16:3	59:15:1	57:15:3	59:15:1
∨	8	8	8	8

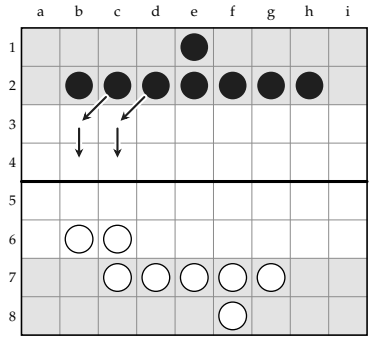
DIAGRAM 58. Black's first turn: c1–b2, d1–c2, f1–g2, g1–h2.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	57:15:3	59:15:1	57:15:3	55:14:1
∨	8	8	8	8

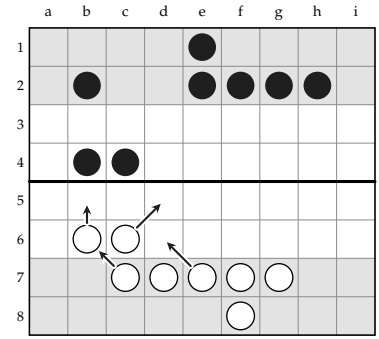
DIAGRAM 59. White's first turn: c7–b6, d7–c6, d8–c7, e8–d7.

Black goes first where White is nearest. Two pearls reach the line together, on b4 and c4. White brings up what it can: pearls on b5 and d5, with two more behind them on b6 and d6.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	57:15:3	55:14:1	53:14:3	55:14:1
∨	8	8	8	8

DIAGRAM 60. Black's second turn: c2–b3, b3–b4, d2–c3, c3–c4.

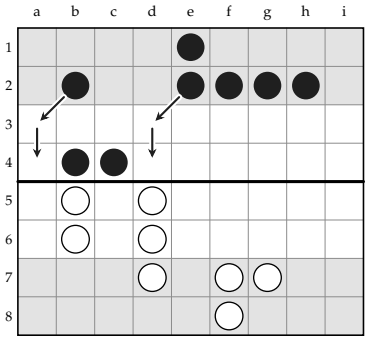


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	53:14:3	55:14:1	53:14:3	51:13:1
∨	8	8	8	8

DIAGRAM 61. White's second turn: b6–b5, e7–d6, c7–b6, c6–d5.

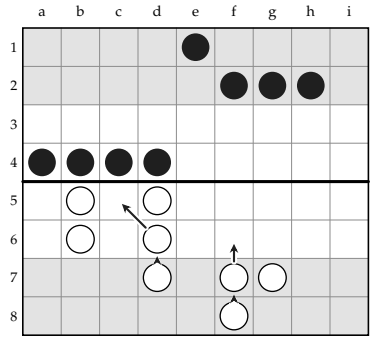
White now threatens the pair. With b6–c5, d5–d4 and d6–d5 the pearl on c4 would have four white pearls round it and one supporting pearl, and when it had gone the pearl on b4 would go too.

Black puts a pearl on each side of the pair. Now b4 and c4 have two supporting pearls each, and it would take six pearls to capture either. Four pearls are on the line and safe. White completes three on its own line, on b5, c5 and d5. Black has nine free steps left and White seven.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	53:14:3	51:13:1	49:13:3	51:13:1
∨	8	8	8	8

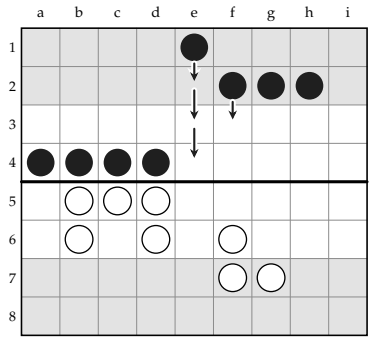
DIAGRAM 62. Black's third turn: b2–a3, a3–a4, e2–d3, d3–d4.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	49:13:3	51:13:1	49:13:3	47:12:1
∨	8	8	8	7

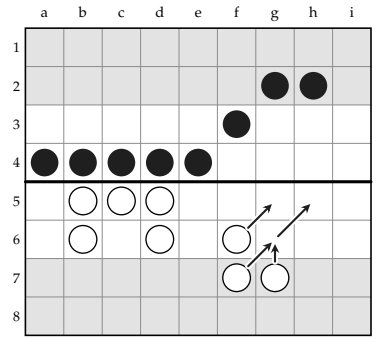
DIAGRAM 63. White's third turn: d6–c5, f7–f6, d7–d6, f8–f7.

The square e4 is next. If Black leaves it, a white pearl can stand there with d5 beside it, and Black's line will stop at d4. The pearl on e1 plays all three of its free steps and takes the square. Five on the line.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	49:13:3	47:12:1	45:12:3	47:12:1
∨	8	7	7	7

DIAGRAM 64. Black's fourth turn: e1–e2, e2–e3, e3–e4, f2–f3.

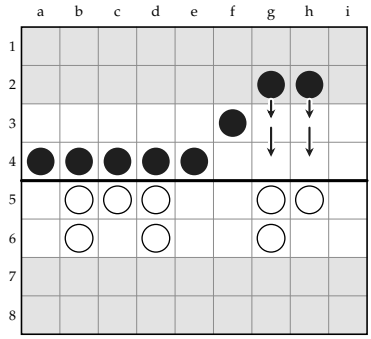


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	45:12:3	47:12:1	45:12:3	43:11:1
∨	7	7	7	6

DIAGRAM 65. White's fourth turn: f6–g5, g7–g6, g6–h5, f7–g6.

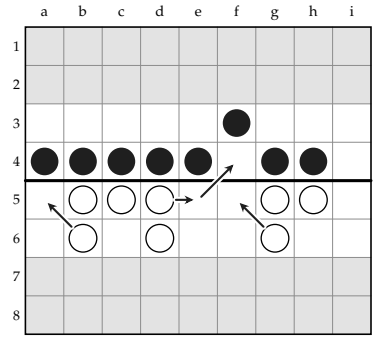
White turns to the other wing, where Black has not yet arrived, and puts pearls on g5 and h5 with a third behind them. Black has five free steps left and White three.

Black has five free steps and can play four. It finishes the end of the line first, on g4 and h4, and leaves f4 for last, with the late pearl on f3 directly behind the gap. The pearl on f3 touches both e4 and g4.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	45:12:3	43:11:1	41:11:3	43:11:1
∨	7	6	6	6

DIAGRAM 66. Black's fifth turn: g2–g3, g3–g4, h2–h3, h3–h4.

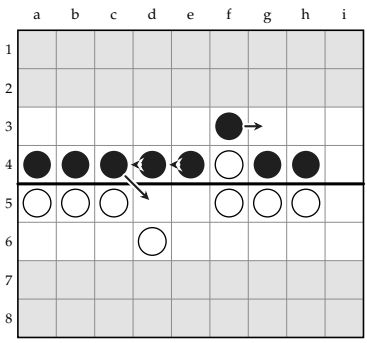


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	41:11:3	43:11:1	41:11:3	40:10:0
∨	6	6	6	6

DIAGRAM 67. White's fifth turn: d5–e5, e5–f4, b6–a5, g6–f5.

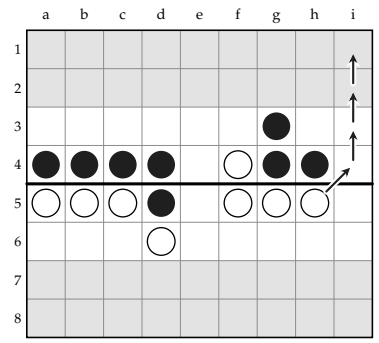
White has three free steps and four to play. This is the spare step. White could complete its line, and then Black would play f3–f4 by the long way round and White would have to wait first. It could play two free steps and keep the third in hand, with the late pearl brought round behind its gap, which Section 11 examines. It does neither. It steps into Black's gap. The pearl from d5 goes by way of e5 to f4, and the pearl from g6 comes to f5 behind it. With f5 and g5 beside it the pearl on f4 cannot be captured. White has gained a row on the f-file. Its count is 40 and Black's 41.

White has weakened its own line by the step. The squares d5 and e5 are empty, with one white pearl behind them on d6. Black steps into d5 with the pearl from c4, and the pearls from d4 and e4 close up behind it, so that the line from a4 to d4 is whole again and the pearl on d5 has two supporting pearls. That is a row for a row, and the counts are level at 40.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	41:11:3	40:10:0	40:10:0	40:10:0
∨	6	6	6	6

DIAGRAM 68. Black's sixth turn: c4–d5, d4–c4, e4–d4, f3–g3.



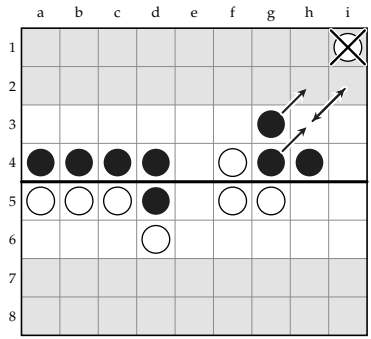
	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	40:10:0	40:10:0	40:10:0	36:9:0
∨	6	6	6	6

DIAGRAM 69. White's sixth turn: h5–i4, i4–i3, i3–i2, i2–i1.

The fourth step matters as much. The late pearl cannot have f4 now. It goes to g3, behind the end of the line, and three black pearls stand together by the open file.

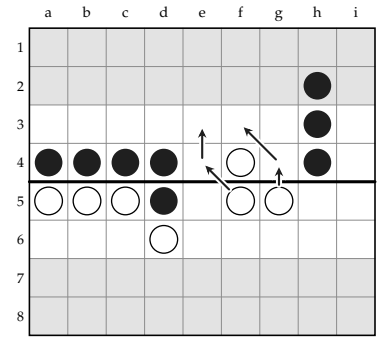
White sends a runner round the end all the same. White 36.

The runner is caught on the last row. Two of the three pearls by the open file go back after it. When they stand on i2 and h2 the runner has two black pearls beside it and no supporting pearl, and it is captured. One of the two then starts forward again.



	BEFORE ●	BEFORE ○	AFTER ●	AFTER ○
+	0	0	0	0
x	0	0	1	0
Σ	40:10:0	36:9:0	35:9:1	35:9:1
∨	6	6	6	6

DIAGRAM 70. Black's seventh turn: g4–h3, h3–i2, g3–h2, i2–h3. The pearl on i1 is captured.



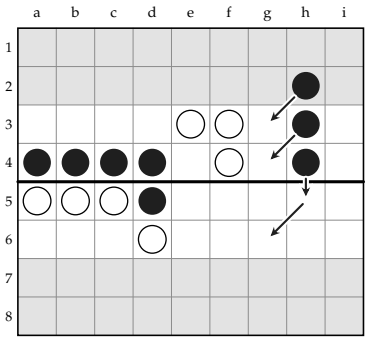
	BEFORE ●	BEFORE ○	AFTER ●	AFTER ○
+	0	0	0	0
x	1	0	1	0
Σ	35:9:1	35:9:1	35:9:1	31:8:1
∨	6	6	6	6

DIAGRAM 71. White's seventh turn: f5–e4, e4–e3, g5–g4, g4–f3.

Black has given up rows to do it. Three of Black's four steps went backward, and the pearls on h2, h3 and h4 are a long way from home. The pearl taken was one step from crossing, and Black is a capture ahead: the target is seven and Black has eight pearls. Both sides need 35 steps.

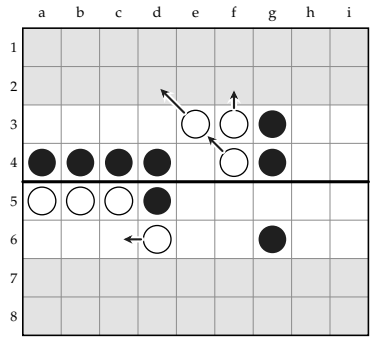
White pushes its center through. The pearls on e3, f3 and f4 are past Black's line with nothing in front of them. White needs 31.

Black cannot stop three pearls with three, and does not try. It sets a runner of its own going, from h4 to g6, and brings the other two pearls back toward the center. White runs on, and its three leading pearls are two, two and three steps from home. Black needs 32 steps and White 28: eight turns against seven.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	1	0	1	0
Σ	35:9:1	31:8:1	32:8:0	31:8:1
∨	6	6	5	6

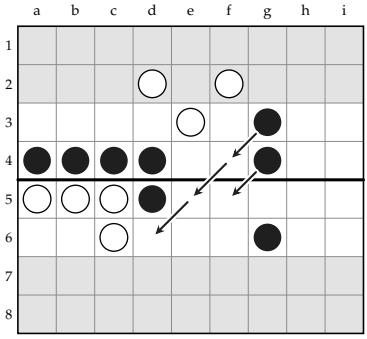
DIAGRAM 72. Black's eighth turn: h3–g4, h4–h5, h2–g3, h5–g6.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	1	0	1	0
Σ	32:8:0	31:8:1	32:8:0	28:7:0
∨	5	6	5	6

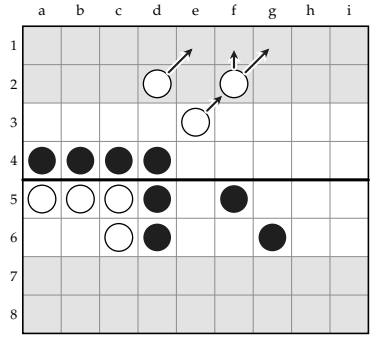
DIAGRAM 73. White's eighth turn: e3–d2, d6–c6, f4–e3, f3–f2.

Black brings two more pearls over the center, to f5 and d6. White puts three pearls on its last row. White needs 24 steps, six turns. Black needs 29, eight turns. On the count White is two turns ahead.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	1	0	1	0
Σ	32:8:0	28:7:0	29:8:3	28:7:0
∨	5	6	5	6

DIAGRAM 74. Black's ninth turn: g4–f5, g3–f4, f4–e5, e5–d6.

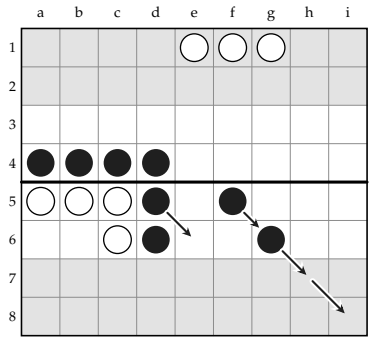


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	1	0	1	0
Σ	29:8:3	28:7:0	29:8:3	24:6:0
∨	5	6	5	6

DIAGRAM 75. White's ninth turn: d2–e1, f2–f1, e3–f2, f2–g1.

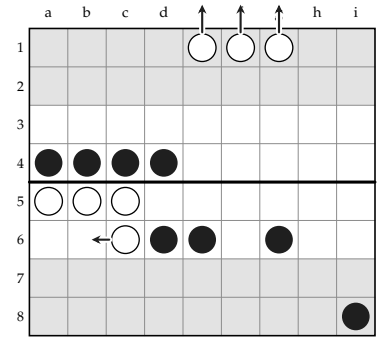
Three of White's pearls are about to score. The other four, on a5, b5, c5 and c6, make up 21 of its 24 steps, and they stand behind the four black pearls on a4, b4, c4 and d4, which have not moved since Black's sixth turn.

Black keeps running and needs 25. White scores three. Its fourth step has to be played by one of the four pearls on the left, and none of them has a forward step. The pearl on c6 could reach d5, but it would stand there beside four black pearls with one supporting pearl. It steps sideways instead. White needs 21 steps, six turns, and has lost a step.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	1	0	1	0
Σ	29:8:3	24:6:0	25:7:3	24:6:0
∨	5	6	5	6

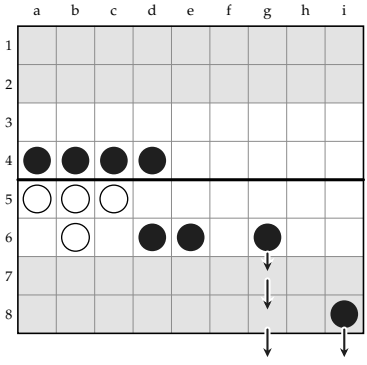
DIAGRAM 76. Black's tenth turn: d5–e6, g6–h7, f5–g6, h7–i8.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	3
x	1	0	1	0
Σ	25:7:3	24:6:0	25:7:3	21:6:3
∨	5	6	5	6

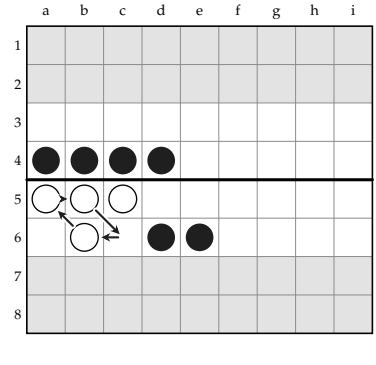
DIAGRAM 77. White's tenth turn: e1–off, f1–off, c6–b6, g1–off.

Black scores two and needs 21. White has no forward step at all. Its four pearls go round in a ring and end on the same four squares: four steps lost, a whole turn. Both sides need 21 steps, six turns, and it is Black's move. The two turns White was ahead by have gone, one step and then four, on the squares in front of Black's wall.



	BEFORE		AFTER	
	●	○	●	○
+	0	3	2	3
x	1	0	1	0
Σ	25:7:3	21:6:3	21:6:3	21:6:3
∨	5	6	5	6

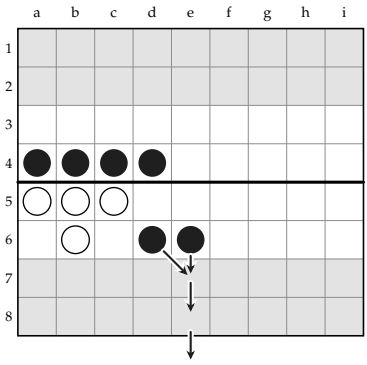
DIAGRAM 78. Black's eleventh turn: g6–g7, g7–g8, g8–off, i8–off.



	BEFORE		AFTER	
	●	○	●	○
+	2	3	2	3
x	1	0	1	0
Σ	21:6:3	21:6:3	21:6:3	21:6:3
∨	5	6	5	6

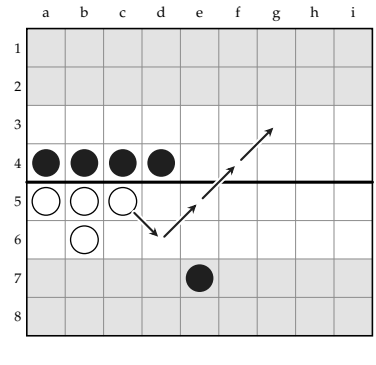
DIAGRAM 79. White's eleventh turn: b5–c6, a5–b5, b6–a5, c6–b6.

Black scores a third pearl and needs 17 steps, five turns. White finds a way out for one pearl. The pearl on c5 goes backward to d6, which Black has just left, and then forward round the end of the wall to g3: one step back and three forward. White needs 19, five turns. Black is to move, and is ahead by the move and no more.



	BEFORE		AFTER	
	●	○	●	○
+	2	3	3	3
x	1	0	1	0
Σ	21:6:3	21:6:3	17:5:3	21:6:3
∨	5	6	5	6

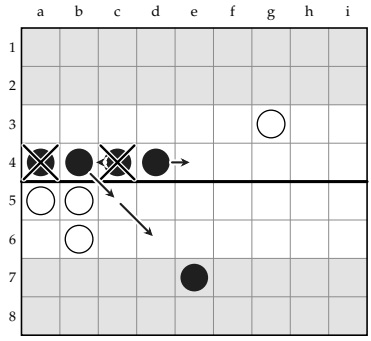
DIAGRAM 80. Black's twelfth turn: e6–e7, e7–e8, e8–off, d6–e7.



	BEFORE		AFTER	
	●	○	●	○
+	3	3	3	3
x	1	0	1	0
Σ	17:5:3	21:6:3	17:5:3	19:5:1
∨	5	6	5	6

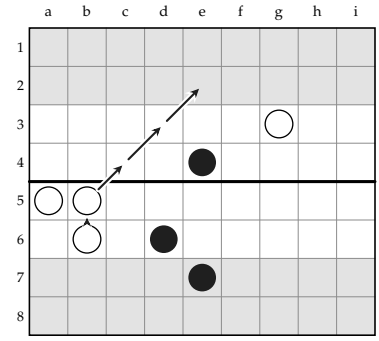
DIAGRAM 81. White's twelfth turn: c5–d6, d6–e5, e5–f4, f4–g3.

Now Black lets the wall go, and gives up two pearls to do it. The pearl on b4 steps forward to c5, and a4, left alone beside two white pearls, is captured. The pearl on c4 steps to b4, where only the pearl on c5 keeps it on the board. That pearl goes on to d6, and b4 is captured as well.



	BEFORE		AFTER	
	●	○	●	○
+	3	3	3	3
x	1	0	1	2
Σ	17:5:3	19:5:1	10:3:2	13:4:3
∨	5	6	5	5

DIAGRAM 82. Black's thirteenth turn: b4–c5, c4–b4, c5–d6, d4–e4. The pearls on a4 and b4 are captured.



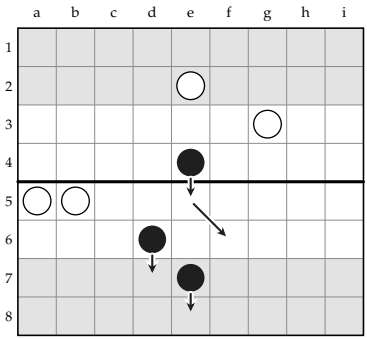
	BEFORE		AFTER	
	●	○	●	○
+	3	3	3	3
x	1	2	1	2
Σ	10:3:2	13:4:3	10:3:2	10:3:2
∨	5	5	5	5

DIAGRAM 83. White's thirteenth turn: b5–c4, c4–d3, d3–e2, b6–b5.

Each of those pearls was five steps from home. With both gone Black has three pearls and needs three crossings: the target is six, and so is Black's maximum. That is ten steps, three turns. White has two captures to Black's one and needs only to reach six, but that is three more crossings from pearls at three, five and five steps: thirteen steps, four turns. Black was ahead by the move. Now it is ahead by a turn.

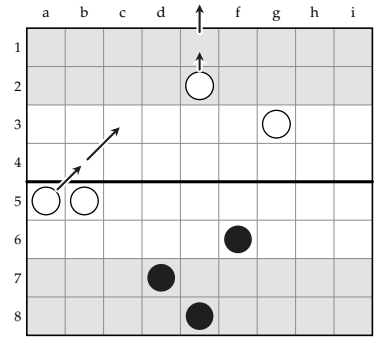
White's pearls are free at last, and one of them runs three rows. White needs ten steps, three turns. So does Black, and Black moves first.

Four forward steps each. Black needs six more steps and White six more. White scores its fourth.



	BEFORE		AFTER	
	●	○	●	○
+	3	3	3	3
x	1	2	1	2
Σ	10:3:2	10:3:2	6:2:2	10:3:2
∨	5	5	3	5

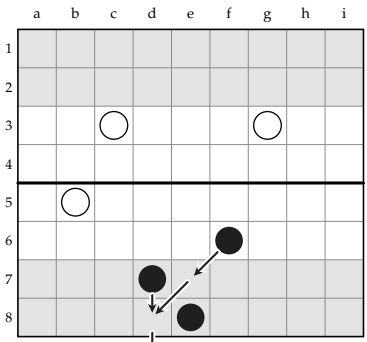
DIAGRAM 84. Black's fourteenth turn: d6–d7, e4–e5, e5–f6, e7–e8.



	BEFORE		AFTER	
	●	○	●	○
+	3	3	3	4
x	1	2	1	2
Σ	6:2:2	10:3:2	6:2:2	6:2:2
∨	3	5	3	3

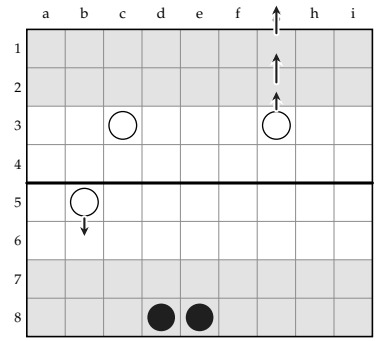
DIAGRAM 85. White's fourteenth turn: e2–e1, e1–off, a5–b4, b4–c3.

Black scores its fourth pearl and puts its last two on the last row. White scores its fifth and needs one more crossing, from the pearl on c3, three steps away. Its other pearl is not needed and steps aside.



	BEFORE		AFTER	
	●	○	●	○
+	3	4	4	4
x	1	2	1	2
Σ	6:2:2	6:2:2	2:1:2	6:2:2
∨	3	3	1	3

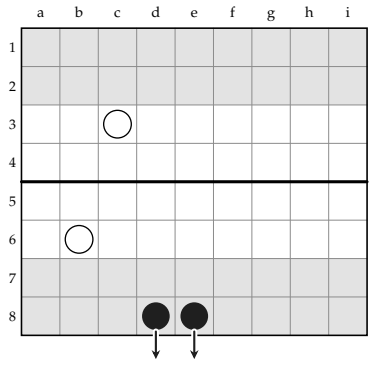
DIAGRAM 86. Black's fifteenth turn: d7–d8, d8–off, f6–e7, e7–d8.



	BEFORE		AFTER	
	●	○	●	○
+	4	4	4	5
x	1	2	1	2
Σ	2:1:2	6:2:2	2:1:2	3:1:1
∨	1	3	1	3

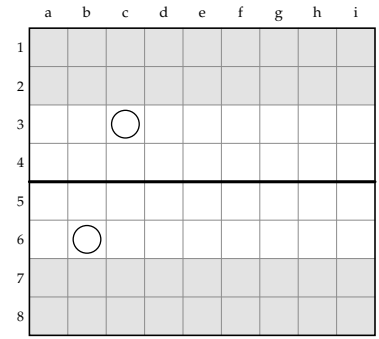
DIAGRAM 87. White's fifteenth turn: b5–b6, g3–g2, g2–g1, g1–off.

Black crosses both pearls and has six. It has no pearls left, so the game is over. White's pearl on c3 was one turn from making it six all, and with two captures to one White would have won the tie.



	BEFORE		AFTER	
	●	○	●	○
+	4	5	6	5
x	1	2	1	2
Σ	2:1:2	3:1:1	0:0:0	3:1:1
∨	1	3	–	3

DIAGRAM 88. Black's sixteenth turn: d8–off, e8–off.



	●	○
+	6	5
x	1	2
Σ	0:0:0	3:1:1
∨	–	3

DIAGRAM 89. The end. Black wins, six to five.

Why Black won. Black came up in the order of Sections 8 and 9. It went first where White was nearest, on the b- and c-files, and arrived there with two pearls together. When White threatened the pair, Black put a pearl on each side of it before a fourth white pearl could come, so that each pearl of the pair had two supporting pearls. It took e4 before White could stand there. It finished the end of its line on g4 and h4 before the middle, and it left its last gap on f4 with the late pearl directly behind it, so that the gap was sound.

White then used its spare step to step into the gap. Black answered in the three parts of Section 12. It stepped into the square the white pearl had left, closed its line behind that pearl, and sent its late pearl to g3, behind the open end of its line. That last placing decided the next turn, because the pearl on g3 was there to catch a runner (Section 19).

From the first capture on, Black played as in Game One. Its pearls on a4, b4, c4 and d4 did not move from its sixth turn to its thirteenth, and White's four pearls on a5, b5, c5 and c6 stood behind them with 21 of White's 24 steps. On its tenth turn White lost a step and on its eleventh a whole turn, and the lead of two turns it had built was gone. On its thirteenth turn Black gave up the two pearls it did not need, on a4 and b4, and finished one turn before White.

Where White went wrong. The step into the gap on White's fifth turn did not lose. It gained a row, and Black needed an exact reply to make the counts level again. White's mistake came on its sixth turn. It sent a runner round the end of Black's line without looking at the black pearl on g3,

which now stood behind the open end. Two black pearls went back after the runner, and it was captured on the last row, one step from crossing. That capture gave Black the spare pearl and the wall. White played well after it. It pushed three pearls through the center and went two turns ahead on the count, and it still lost those two turns in front of Black's wall. White's pearl on c3 was one turn from making the score six all, and with two captures to one White would then have won.

23. The Main Points

1. In a level lock the counts are equal and tell nothing. What matters is which side has to open it.
2. The side that opens a level lock loses. The only opening that does not lose a pearl at once is a runner round the open end, and it gives the other side the square to answer through.
3. A turn that brings a side's count to a new low is a forward turn. Any other turn is a waiting turn. When both sides have waited, the side that waited first must play a forward turn.
4. The side that plays the last free step takes the lock. More steps in hand wins. With a step each, the side to move loses.
5. If both sides come straight up, the side that started ahead runs out of free steps first. The lock goes to the side behind: Black in A, D and E, White in B, C, F, G, H and I.
6. In A, C, E, G, H and I the side ahead has no spare step, and the side behind takes the lock by coming up in good order. In B, D and F the side ahead has one spare step, and the lock is in dispute.
7. Count both sides' free steps before every turn of the walk.
8. Go first where the opponent is nearest. Arrive in pairs, and give the pair a third pearl before a fourth attacker can come.
9. Finish the ends of the line first. Leave the last gap inside the line with the late pearl directly behind it.
10. When the late pearl stands to one side of its gap, the other side can step into the gap and capture the pearls beside it. A late pearl two rows back can be captured.

11. A side with a gap inside a complete line loses a pearl with every step the other side plays.
12. Answer the first runner at once, through the square it left. Then wait. Do not send a second runner.
13. When a line becomes shorter than the line facing it, its end pearl has three enemy pearls in front of it, and a runner behind makes four.
14. After the first capture, the far end of your line is a wall you do not have to move, and the opponent has pearls behind it that must get home.
15. Let the wall go when the count without it is a turn better than the opponent's.
16. A pearl behind the open end of the line catches a runner on the last row.

The Waiting Game

A way of playing without the edge, and where it breaks

In every arrangement one side begins without the edge on the count. In arrangements A, D and E that side is Black, and in B, C, F, G, H and I it is White. The usual advice to the side without the edge is to race anyway and to make up the difference with captures. This article examines a different plan. The side without the edge does not race. It keeps all eight of its pearls in its first three rows, lets the opponent come to it, captures the pearls that the opponent sends forward carelessly, and scores at the end of the game. We call this plan the *waiting game*.

The article finds that the waiting game succeeds against an opponent who sends pearls forward alone or leaves them where they cannot go forward. It fails against an opponent who attacks the end of the waiting side's line with two pearls together, keeps its other pearls able to cross, and gives up only the pearls it does not need. The two games in Part Four show one result of each kind.

Part One sets out how the waiting side arranges its pearls and how it plays its turns without moving forward. Part Two discusses where the opponent should attack. Part Three discusses when the waiting side should go forward and how it should play the ending. Part Four gives two complete games. Part Five sets out the conclusions and the questions that remain open.

Part One

The Wall

1. What the Waiting Side Needs

The waiting side stands its pearls in a line across its third row. We call that line the *wall*. A wall on the third row closes the squares in front of it to any white pearl that arrives alone, because each of those squares touches two black pearls.

A side that plays the waiting game must be able to do five things. It must play its four steps on every turn without weakening the wall. It must arrange eight pearls across nine files. It must know where the opponent will attack the wall and what it will lose to that attack. It must know when to leave the wall and go forward, which we call *coming out*. And it must finish the game by pinning the opponent's last pearls without taking them too soon, because a capture that leaves the opponent with no pearls ends the game. Sections 2 to 6 discuss the five in that order.

2. Marking Time

The waiting side must still move. Every turn is four steps, and no pearl may end its turn on the square where it began. If the wall had to move forward a little on every turn, the waiting side would soon be racing after all.

Diagram 1 shows how the waiting side plays a turn without changing its position. The pearl on b3 steps back to b2 and across to a2. The pearl on a3 steps to b3. The first pearl then steps up to a3. Both pearls have moved, neither ends on the square where it began, and the wall stands as it stood before the turn. The square b3 is empty only during Black's own turn, so White can never use it.

The waiting side can play this turn wherever two pearls of the wall stand side by side with an empty square behind them. It needs that empty square, and that is why the wall stands on the third row and not the second. A turn of this kind is called *marking time*. The waiting side loses no pearl and no part of its position by marking time, and it can mark time

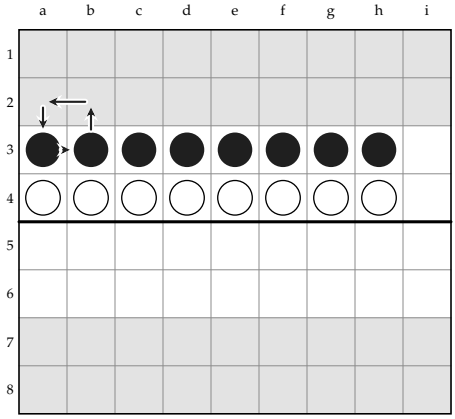


DIAGRAM 1. Black to play. Two pearls change places, and the wall is as it was: b3–b2, b2–a2, a3–b3, a2–a3.

for as many turns as it likes. The opponent therefore cannot win simply by waiting for the wall to break up.

Marking time can also move the wall sideways. A pearl that steps back and comes up again one file along has moved the line by one file. Section 3 discusses why the waiting side needs to do this.

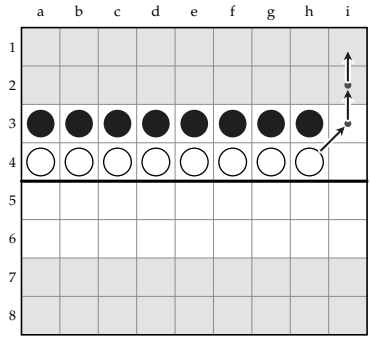
3. Eight Pearls for Nine Files

The wall has eight pearls and the board has nine files, so one file is always open. Most of the waiting side's choices concern where that open file should be.

The open file at the edge. The simplest wall is a line of eight pearls from one edge of the board, as in Diagram 2. The ninth file is then open at the other edge. A white pearl that reaches the square beside it crosses in one turn, and once both sides have formed their lines, Black cannot prevent that crossing.

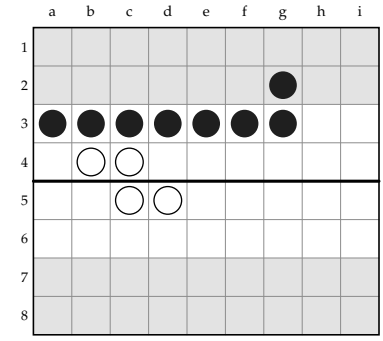
A single crossing would not matter if White then had to go on attacking the wall. White does not have to. In Diagram 3 White has crossed once, has four pearls left and leads by one point. Black is waiting for White to attack, and White is waiting for Black to come out. Neither side has a reason to move first, so the waiting side that gives away one crossing lets the opponent wait as well, with the lead.

Seven and one. The first remedy is to make the line one pearl shorter and to put the eighth pearl behind its end. Seven pearls stand from the a-file



	●	○
+	0	0
x	0	0
Σ	48:12:0	32:8:0
∨	6	4

DIAGRAM 2. White to play. The pearl on h4 crosses on this turn by the open i-file.



	●	○
+	0	1
x	3	0
Σ	30:8:2	18:5:2
∨	6	5

DIAGRAM 3. A later position of the same kind. White leads by one point and does not attack.

to the g-file, and the eighth stands on g2. We call it the *back pearl*, and we call the whole arrangement *seven and one*.

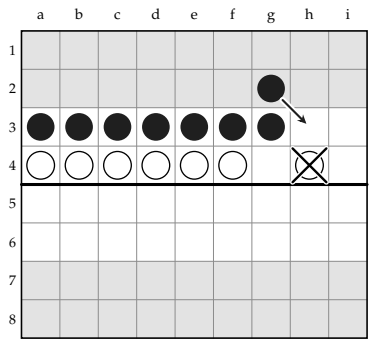


DIAGRAM 4. Black to play. The back pearl takes a single white pearl beside the open files.

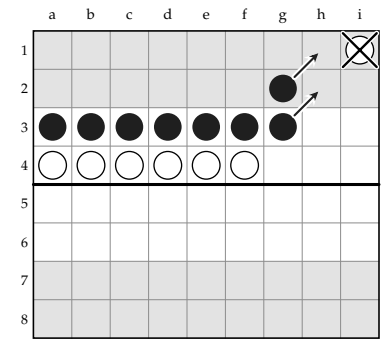


DIAGRAM 5. Black to play. A runner that reaches the back row is taken there.

The back pearl has three uses. It gives the end pearl of the line a second supporting pearl. It captures a single white pearl that stops beside the open files, as in Diagram 4. And it captures a single white pearl that tries to run past the end of the line. A white pearl starting from the fifth row needs five steps to cross by the i-file, so it must stop on the way at the end of a turn, and wherever it stops alone, the back pearl and the end pearl

can take it. In Diagram 5 the runner has reached the back row. The end pearl steps to h2, the back pearl steps to h1, and the runner is captured. The pearl on h1 then returns to the wall.

Seven and one therefore stops every white pearl that comes forward alone. Section 4 explains why it does not stop two white pearls that come together.

The open file in the middle. The second remedy is to put the open file inside the line, as in Diagram 6. We call it the *gap*. Both edge files are then covered, and no white pearl can pass through the gap, however many supporting pearls it has. A white pearl can step into the gap square itself. Every square behind the gap, however, touches a black pearl on each side, and a white pearl that steps there has left its supporting pearls behind. It would stand alone beside two black pearls, so it may not make that step. A gap one file wide inside the line stays closed for as long as the two pearls beside it stand.

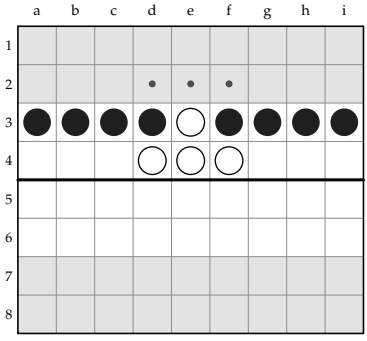
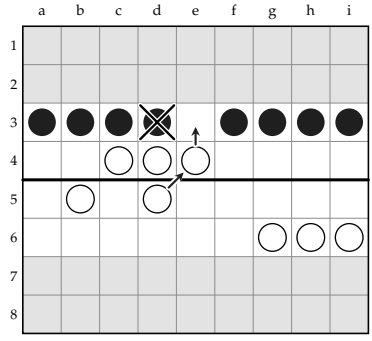


DIAGRAM 6. White to play. The white pearl on e3 cannot pass through the gap.



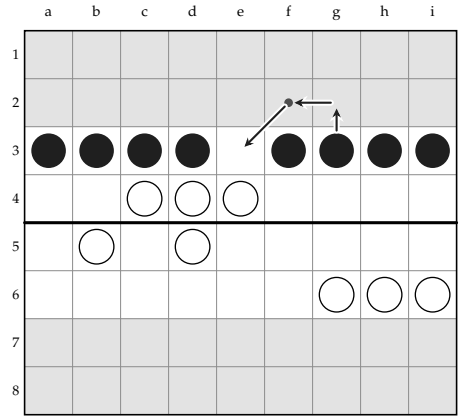
	●	○
+	0	0
x	0	0
Σ	48:12:0	40:10:0
∨	6	6

DIAGRAM 7. White to play. Four white pearls take the black pearl beside the gap.

The weakness of the gap is that each of the two pearls beside it has only one supporting pearl, so four white pearls can capture either of them: three in front of it and one in the gap. Diagram 7 is from the second game in Part Four. White has three pearls in front of the black pearl on d3 and needs two more steps to capture it.

Moving the gap. Black's answer is to move the gap away. In Diagram 8 it is Black's turn in the same position. One black pearl goes from g3 to e3, the gap moves to the g-file, where White has nothing in front of it, and

White can no longer capture. Black moves the gap one file for each step it plays. Each of the three or four white pearls that threaten the gap must also move a file to follow it, so a waiting side that watches for white pearls gathering in front of its gap can always move the gap away before they arrive.



	●	○
+	0	0
x	0	0
Σ	48:12:0	40:10:0
∨	6	6

DIAGRAM 8. Black to play in the same position. Black moves the gap from the e-file to the g-file.

This holds while White's pearls stand in groups. When White has a full line on its fourth row, there are three white pearls in front of every file, and the gap has nowhere to go. The waiting side must then put its open file at the edge again, form seven and one, and meet the attack described in Part Two.

Part Two

The Attack on the Wall

4. Where the Wall Is Attacked

A pearl in the middle of the wall has a supporting pearl on each side, and the opponent cannot bring enough pearls round it from the front to capture it. A good opponent therefore attacks the end of the wall.

Four against the end. The last pearl of a line has one supporting pearl, so four white pearls capture it. In Diagram 9 the pearl on f3 is the end of a line of four, with the gap beside it. White needs pearls on e4, f4, g4 and g3. Two are already there, and the other two arrive this turn and take the pearl. The back pearl prevents this attack. With a black pearl on g2, the end pearl on g3 has two supporting pearls, six white pearls would be needed round it, and White cannot place six round it from the front.

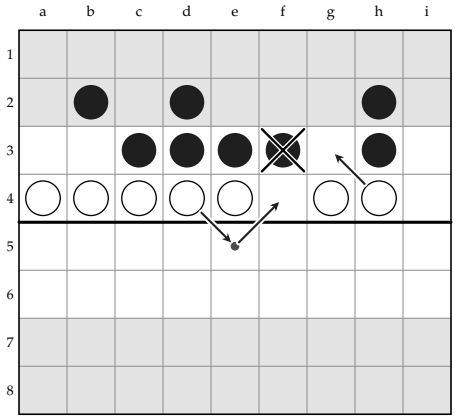
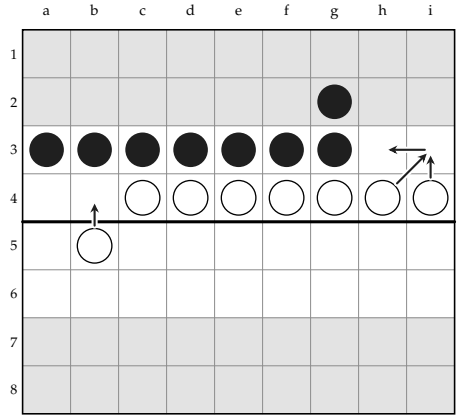


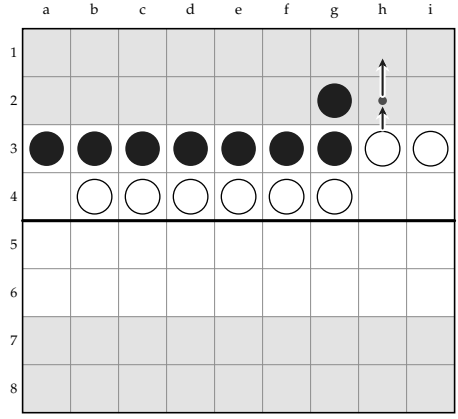
DIAGRAM 9. White to play. Four white pearls against the end of a line.

Two pearls beside the end. Seven and one does not meet the attack of Diagram 10, which is from the second game in Part Four. Two white pearls step together onto h3 and i3, in the open files beside the end of the line. The pearl on h3 touches both the end pearl and the back pearl, but it has



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	49:13:3	33:9:3	49:13:3	30:8:2
∨	7	5	7	4

DIAGRAM 10. White to play: i4–i3, i3–h3, b5–b4, h4–i3. Two white pearls land together beside the end of the line.



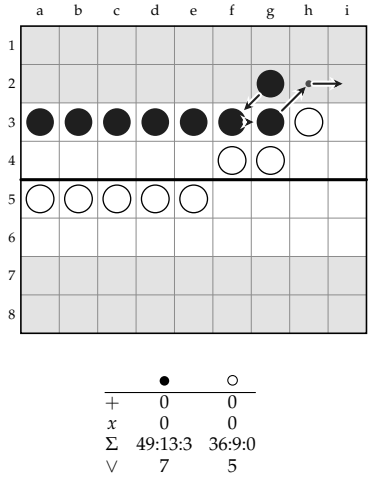
	●	○
+	0	0
x	0	0
Σ	49:13:3	30:8:2
∨	7	4

DIAGRAM 11. Black to play. Every turn that stops the pearl on h3 from crossing opens the wall elsewhere.

a supporting pearl on i3, so four black pearls would be needed to take it, and Black cannot bring four round it.

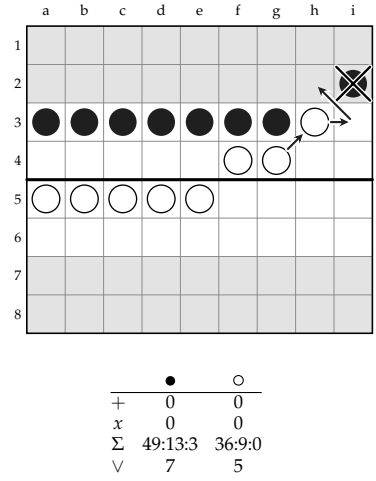
Diagram 11 shows the position after that turn, with Black to play. Nearly every turn Black can play lets a white pearl cross on White's next turn. The few turns that prevent it each take three of the seven black pearls off the third row, and that opens the wall in other places. In practice Black has no defense. In the game Black marked time, White crossed, and White went on crossing by the same route until the game was over.

One pearl with a supporting pearl. A single white pearl beside the end of the line does the same if a second white pearl supports it from g4. Diagram 12 shows the position. If Black does nothing, the pearl on h3 crosses on White's next turn by i2 and i1. Few of Black's turns stop a white pearl from crossing on that turn, and only one of them keeps all seven black pearls on the third row. It is the turn shown. The end pearl goes to i2 by way of h2, and the two pearls behind it each move up one place. The line is still seven pearls long, and the eighth pearl now stands on i2, where it shuts h2 to the white pearl on h3.



	●	○
+	0	0
x	0	0
Σ	49:13:3	36:9:0
∨	7	5

DIAGRAM 12. Black to play. One white pearl stands beside the end of the line, supported from g4.



	●	○
+	0	0
x	0	0
Σ	49:13:3	36:9:0
∨	7	5

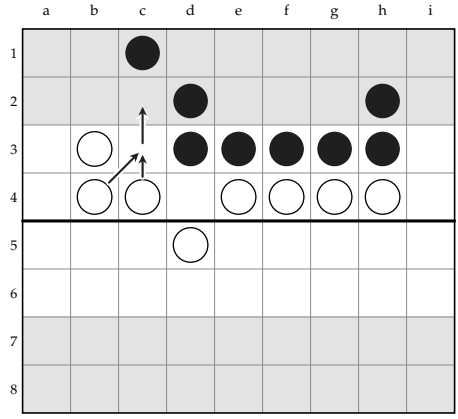
DIAGRAM 13. White to play. White takes the pearl on i2 and puts a pearl on h2.

That defense lasts for one turn. In Diagram 13 White takes the pearl on i2 with two steps and puts a pearl on h2 with the third. After that, Black has no turn that stops a white pearl from crossing.

So the waiting side cannot hold the end of its wall, and the reason is not peculiar to seven and one. No arrangement of eight pearls in the first

three rows both closes every square to a lone pearl and leaves no pearl that four enemy pearls can surround. Every wall of eight either leaves a way through or has a pearl that four white pearls can take.

A wall that draws back. The waiting side does worse if it moves its wall away from an attack. In Diagram 14 White has gathered three pearls opposite Black's open a-file, and Black has drawn its line away from them. Now the a-file and the b-file are both open, and the three white pearls walk through together. All three crossed before Black made a capture. When white pearls gather opposite the open end of the wall, the waiting side should keep its wall where it is and put its back pearl at that end.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	52:13:0	32:8:0	52:13:0	29:8:3
∨	8	5	8	5

DIAGRAM 14. White to play: c4–c3, c3–c2, b4–c3. Black has drawn its wall away and left two files open.

What follows from the attack. The waiting side should expect the opponent to cross at the end of its wall. The waiting game can still succeed after those crossings, but only if the opponent loses pearls in making them, or leaves its remaining pearls where they cannot go forward. In the first game of Part Four White crosses three times by this route and loses the game, because it lost five pearls and Black played the ending correctly. In the second game White crosses four times and loses only pearls it chose to give up. Part Four explains the difference between the two games.

Part Three

Coming Out and the Ending

5. When to Come Out

The waiting side cannot wait for the whole game. It must decide when to leave the wall and go forward, and this is the one decision in the waiting game that requires counting.

It should come out when either of two things has happened. The first is that the opponent has three pearls or fewer on the board. The second is that the opponent has scored twice, however many pearls it has left.

> *Come out when the opponent has three pearls or fewer, or has scored twice.*

The first rule applies when the plan has worked. The opponent has lost its pearls against the wall, and three pearls cannot stop eight from crossing.

The second rule applies when the plan has failed. An opponent with two points can stop attacking, or can end the game by crossing or giving up its remaining pearls. In either case the wall no longer gains the waiting side anything. The waiting side needs crossings of its own, and it loses a turn for every turn it waits.

Diagram 15 shows the first rule. White has brought a single pearl to h4. The back pearl takes it, and Black uses the other three steps of its turn to start forward. Black does not send one pearl ahead on its own. The pearls go forward together, so that none of them can be taken.

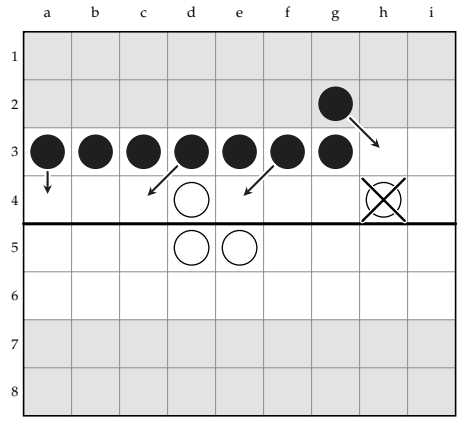


DIAGRAM 15. Black to play. A capture leaves White with three pearls, and Black's wall goes forward.

6. The Ending

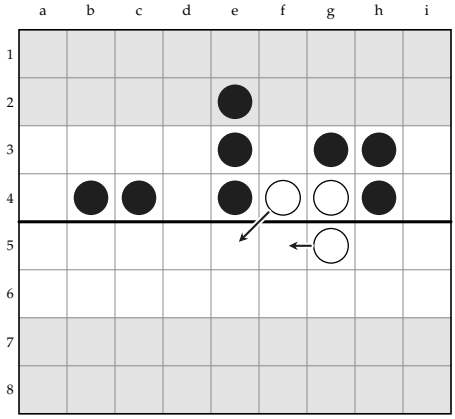
At the end of a waiting game the waiting side is usually behind on points, ahead on captures, and still has most of its pearls. The opponent has one, two or three pearls left. From here the game depends on one rule: the game ends as soon as either side has no pearl on the board.

The opponent wants the game to end. In Diagram 16 White has scored twice and Black has not scored. White walks two pearls away and leaves the third to be taken. A turn later it gives up another and crosses with the last. Black finished with all eight pearls on the board, five captures and no points. Each capture Black made in this ending brought the end of the game nearer while White was ahead, so each of them helped White.

Pinning the last pearls. A pearl that stands alone may not step onto a square that touches two enemy pearls. If every square beside it touches two, it has no legal step. We call such a pearl *pinned*. A side whose pearls on the board are all pinned loses its turn, and it goes on losing its turns for as long as they stay pinned.

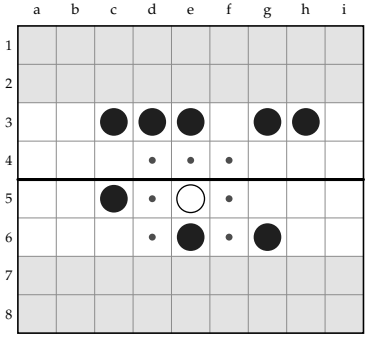
In Diagram 17 Black is two points behind and White has one pearl. Black has not taken it. Three black pearls have gone out round it, the rest of the wall stands behind, and the white pearl has no legal step. White lost four turns in a row while Black walked two pearls forward and across. With the score level, Black made the capture in Diagram 18 and won on captures.

It takes five pearls to pin a pearl in the open board, three to pin it against an edge, and two to pin it in a corner. When the waiting side cannot pin



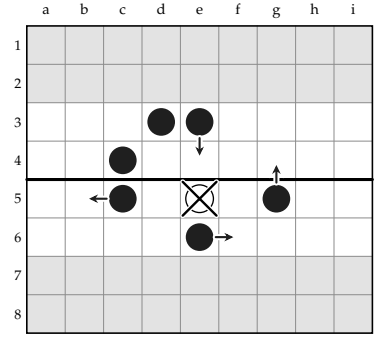
	BEFORE		AFTER	
	●	○	●	○
+	0	2	0	2
x	3	0	3	0
Σ	26:7:2	13:4:3	26:7:2	14:4:2
∨	6	5	6	5

DIAGRAM 16. White to play, two points ahead. White gives up a pearl and runs.



	●	○
+	0	2
x	5	0
Σ	10:3:2	5:2:3
∨	4	5

DIAGRAM 17. White to play. Black is two points behind. The white pearl is pinned, and White loses its turn.



	●	○
+	2	2
x	5	0
Σ	3:1:1	5:2:3
∨	3	5

DIAGRAM 18. Four turns later, with the score level, Black takes the pearl.

the last pearl, it should leave it alone. The first game in Part Four ends that way. Black is three points behind, and White has one pearl on the board.

Black leaves that pearl alone for seven of its turns, scores four points, and only then takes it.

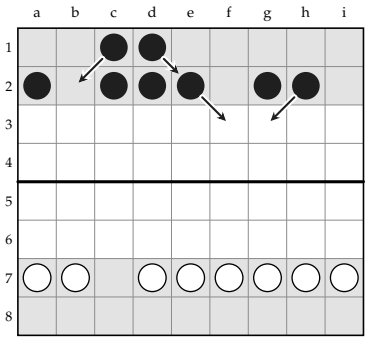
> *Before any capture that would leave the opponent with no pearls, check the score.*

Part Four
Two Games

7. Two Games

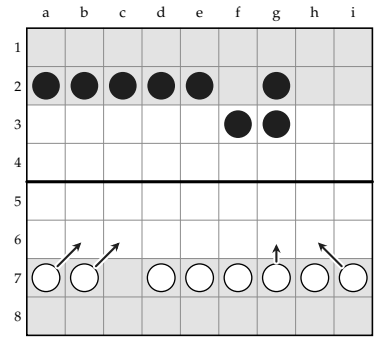
Both games begin from arrangement A, in which White has all eight pearls on its front row and Black has six on its front row. White's count is 56 and Black's is 58, so White has the edge, and in both games Black plays the waiting game. Each diagram shows the position before a turn, with the steps of that turn marked, and the table beneath it gives the figures before and after the turn.

First game. White crosses three times at the end of Black's wall, but it loses five pearls, and its last pearls are left in front of the wall where they cannot go forward. Black wins by four points to three.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	58:15:2	56:14:0	54:14:2	56:14:0
∨	8	7	7	7

DIAGRAM 19. Black 1. e2–f3, h2–g3, d1–e2, c1–b2.

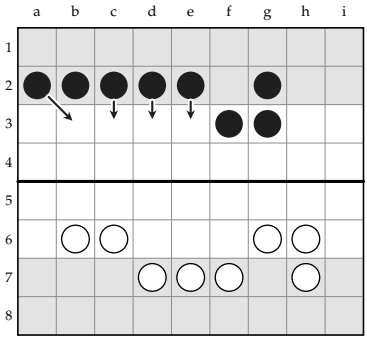


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	54:14:2	56:14:0	54:14:2	52:13:0
∨	7	7	7	7

DIAGRAM 20. White 1. b7–c6, g7–g6, a7–b6, i7–h6.

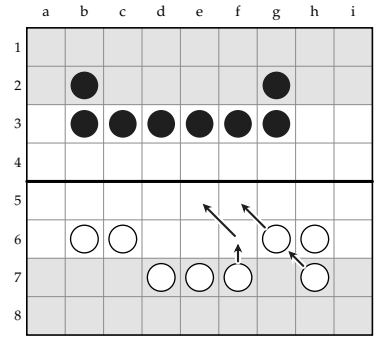
Black 1. Black starts building its wall on the third row. Its two back pearls come up behind, so that no black pearl stands alone.

White 1. White comes forward on both wings. White has the edge on the count and needs only to keep it.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	54:14:2	52:13:0	50:13:2	52:13:0
∨	7	7	7	7

DIAGRAM 21. Black 2. d2–d3, a2–b3, c2–c3, e2–e3.

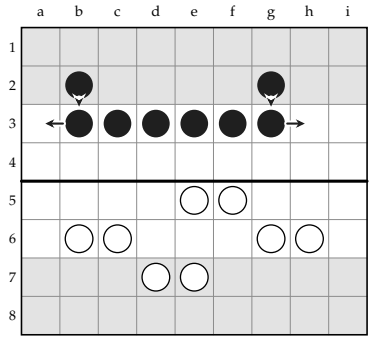


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	50:13:2	52:13:0	50:13:2	48:12:0
∨	7	7	7	7

DIAGRAM 22. White 2. g6–f5, f7–f6, f6–e5, h7–g6.

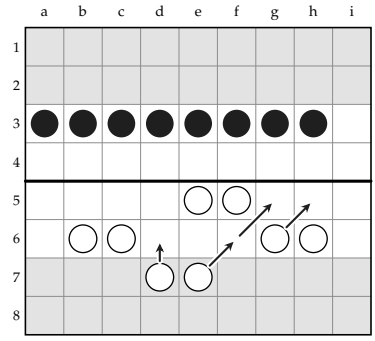
Black 2. Six black pearls now stand on the third row, with one behind each end.

White 2. White brings up its center.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	50:13:2	48:12:0	48:12:0	48:12:0
∨	7	7	6	7

DIAGRAM 23. Black 3. b3–a3, g3–h3, g2–g3, b2–b3.

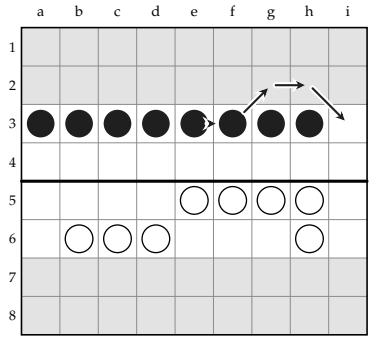


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	48:12:0	48:12:0	48:12:0	44:11:0
∨	6	7	6	6

DIAGRAM 24. White 3. g6–h5, e7–f6, f6–g5, d7–d6.

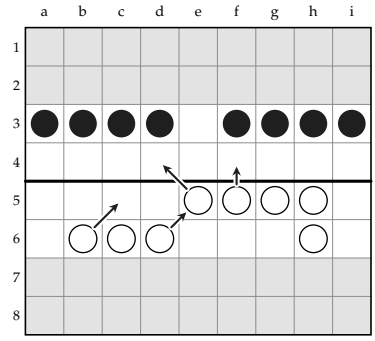
Black 3. Black completes a line of eight from the a-file to the h-file. The i-file is open, and Section 3 explains why an open file at the edge cannot be held.

White 3. White gathers its pearls on its right, toward the open i-file.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	48:12:0	44:11:0	48:12:0	44:11:0
∨	6	6	6	6

DIAGRAM 25. Black 4. f3–g2, g2–h2, h2–i3, e3–f3.

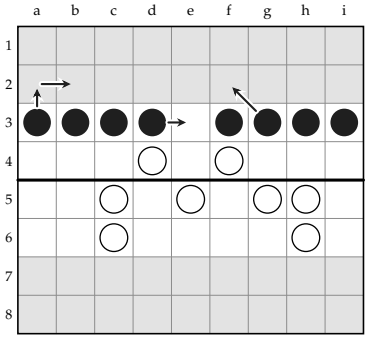


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	48:12:0	44:11:0	48:12:0	40:10:0
∨	6	6	6	6

DIAGRAM 26. White 4. f5–f4, b6–c5, e5–d4, d6–e5.

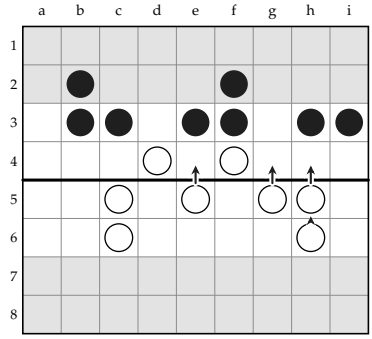
Black 4. Black moves the open file into the middle of its line. The pearl from f3 goes round behind the line to i3, and the pearl on e3 steps across to f3. The gap is now on the e-file, and no white pearl can pass through it. This is the right answer to white pearls gathering opposite the open end.

White 4. White puts pearls on d4 and f4, on either side of the gap.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	48:12:0	40:10:0	50:13:2	40:10:0
∨	6	6	7	6

DIAGRAM 27. Black 5. d3–e3, a3–a2, a2–b2, g3–f2.

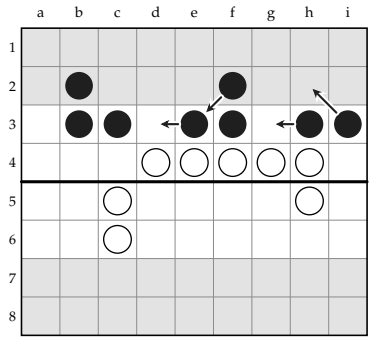


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	50:13:2	40:10:0	50:13:2	36:9:0
∨	7	6	7	6

DIAGRAM 28. White 5. h5–h4, g5–g4, e5–e4, h6–h5.

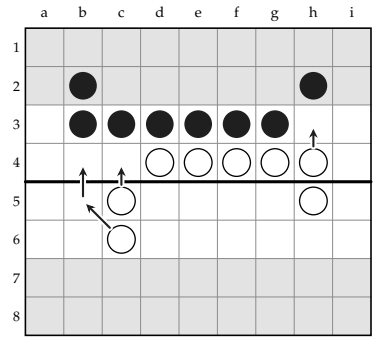
Black 5. Black closes the e-file and draws two pearls back to the second row. After this turn the a-, d- and g-files are all open on the third row. A wall with three open files gives White three places to attack.

White 5. White forms a line of five on its fourth row, from d4 to h4.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	50:13:2	36:9:0	50:13:2	36:9:0
∨	7	6	7	6

DIAGRAM 29. Black 6. e3–d3, i3–h2, h3–g3, f2–e3.

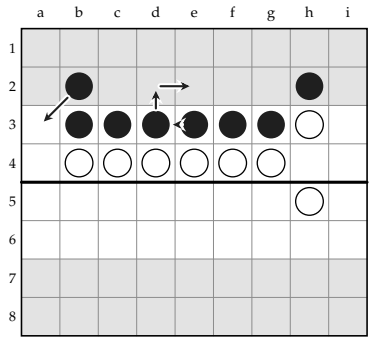


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	50:13:2	36:9:0	50:13:2	32:8:0
∨	7	6	7	5

DIAGRAM 30. White 6. c6–b5, b5–b4, h4–h3, c5–c4.

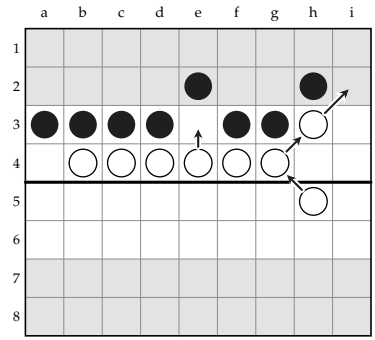
Black 6. Black closes up again: six pearls in line from b3 to g3, with a pearl behind each end on b2 and h2.

White 6. White puts a pearl on h3, beside the end of Black's line, with a pearl on g4 behind it in support. This is the attack of Diagram 12, in Section 4. From here Black cannot stop White from crossing at the end of its wall.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	50:13:2	32:8:0	50:13:2	32:8:0
∨	7	5	7	5

DIAGRAM 31. Black 7. d3–d2, d2–e2, b2–a3, e3–d3.

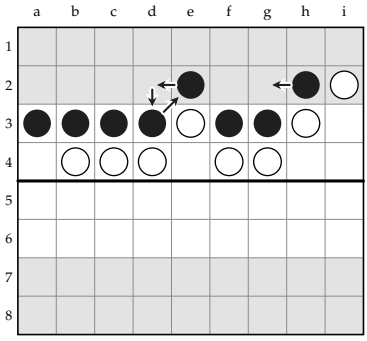


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	50:13:2	32:8:0	50:13:2	28:7:0
∨	7	5	7	4

DIAGRAM 32. White 7. h3–i2, g4–h3, e4–e3, h5–g4.

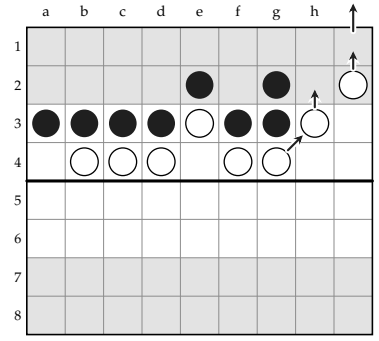
Black 7. Black moves the left end of its line to the edge and draws the pearl on e3 back to e2, which opens a gap on the e-file. As Section 4 explains, Black has no turn that stops the pearl on h3.

White 7. The pearl on h3 steps to i2, and a second pearl takes its place on h3. A third white pearl steps into the gap on e3. That pearl is safe for the moment, because the white pearls on d4 and f4 support it, but it cannot go on through the wall. This is White's first mistake of the game: the pearl is now tied to the pearls beside it, and Black takes it at its eleventh turn, once one of them has moved on.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	50:13:2	28:7:0	50:13:2	28:7:0
∨	7	4	7	4

DIAGRAM 33. Black 8. e2–d2, h2–g2, d3–e2, d2–d3.

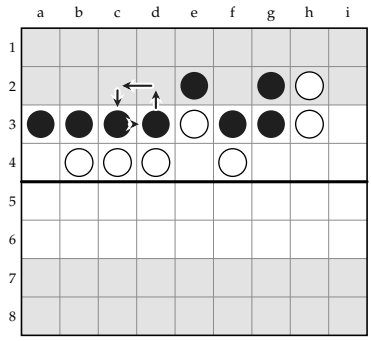


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	1
x	0	0	0	0
Σ	50:13:2	28:7:0	50:13:2	24:6:0
∨	7	4	7	4

DIAGRAM 34. White 8. i2–i1, h3–h2, g4–h3, i1–off.

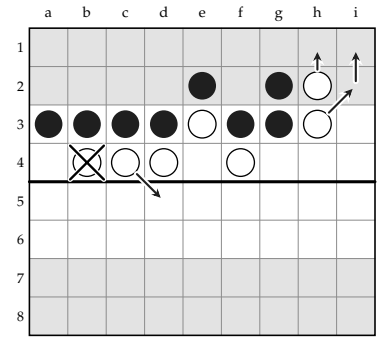
Black 8. Black marks time in the center and moves a pearl to g2, behind the end of its line.

White 8. White crosses for the first time, by the i-file, and keeps two pearls on h2 and h3. White leads by one point.



	BEFORE		AFTER	
	●	○	●	○
+	0	1	0	1
x	0	0	0	0
Σ	50:13:2	24:6:0	50:13:2	24:6:0
∨	7	4	7	4

DIAGRAM 35. Black 9. d3–d2, d2–c2, c3–d3, c2–c3.

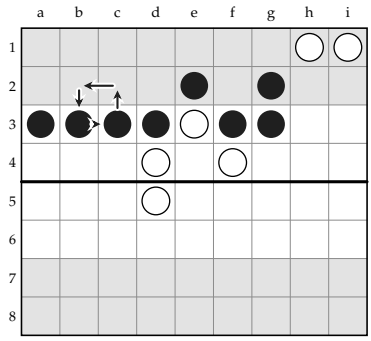


	BEFORE		AFTER	
	●	○	●	○
+	0	1	0	1
x	0	0	1	0
Σ	50:13:2	24:6:0	43:11:1	18:5:2
∨	7	4	7	5

DIAGRAM 36. White 9. c4–d5 ×b4, h3–i2, h2–h1, i2–i1.

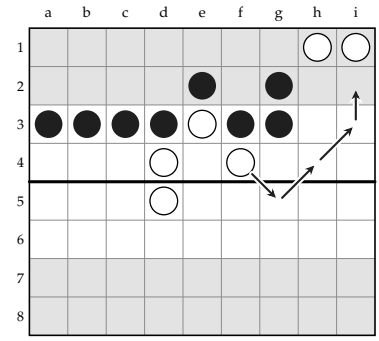
Black 9. Black marks time.

White 9. White takes its two leading pearls to h1 and i1 and gives up the pearl on b4, which stood alone beside three black pearls. White did not need that pearl. Giving it up lowers the target to seven and White's count from 24 to 18.



	BEFORE		AFTER	
	●	○	●	○
+	0	1	0	1
x	1	0	1	0
Σ	43:11:1	18:5:2	43:11:1	18:5:2
∨	7	5	7	5

DIAGRAM 37. Black 10. c3–c2, c2–b2, b3–c3, b2–b3.

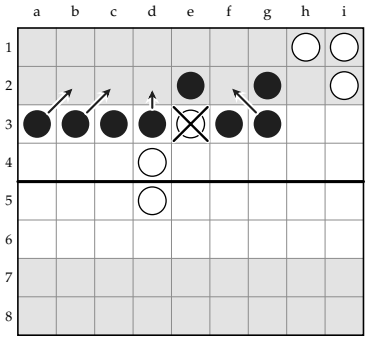


	BEFORE		AFTER	
	●	○	●	○
+	0	1	0	1
x	1	0	1	0
Σ	43:11:1	18:5:2	43:11:1	16:4:0
∨	7	5	7	5

DIAGRAM 38. White 10. f4–g5, g5–h4, h4–i3, i3–i2.

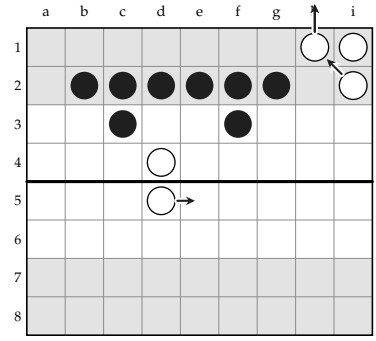
Black 10. Black marks time. The white pearls on h1 and i1 support each other, so four black pearls would be needed to take either of them.

White 10. The pearl from f4 comes round to i2. It leaves the white pearl on e3 with only one supporting pearl, on d4.



	BEFORE		AFTER	
	●	○	●	○
+	0	1	0	1
x	1	0	2	0
Σ	43:11:1	16:4:0	40:10:0	13:4:3
∨	7	5	7	5

DIAGRAM 39. Black 11. b3–c2, a3–b2, g3–f2 ×e3, d3–d2.

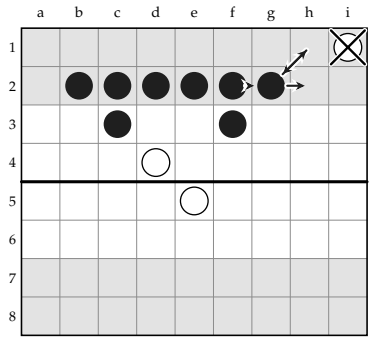


	BEFORE		AFTER	
	●	○	●	○
+	0	1	0	3
x	2	0	2	0
Σ	40:10:0	13:4:3	40:10:0	10:3:2
∨	7	5	7	5

DIAGRAM 40. White 11. d5–e5, h1–off, i2–h1, h1–off.

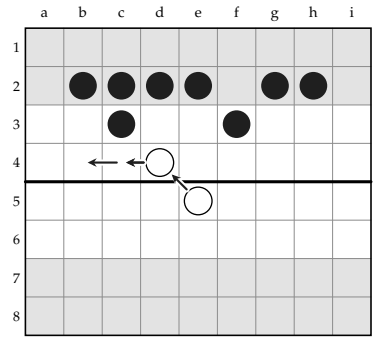
Black 11. Black drops its line to the second row and takes the white pearl on e3.

White 11. Two more white pearls cross, and White leads by three points to none. This is the turn on which White begins to lose the game. It leaves its pearl on i1 alone in the corner, one step from crossing, and its last two pearls, on d4 and e5, stand in front of Black's wall, four and five steps from crossing.



	BEFORE		AFTER	
	●	○	●	○
+	0	3	0	3
x	2	0	3	0
Σ	40:10:0	10:3:2	33:9:3	9:3:3
∨	7	5	7	5

DIAGRAM 41. Black 12. g2–h2, f2–g2, g2–h1 ×i1, h1–g2.

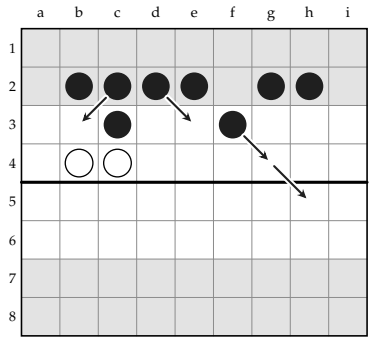


	BEFORE		AFTER	
	●	○	●	○
+	0	3	0	3
x	3	0	3	0
Σ	33:9:3	9:3:3	33:9:3	8:2:0
∨	7	5	7	4

DIAGRAM 42. White 12. d4–c4, c4–b4, e5–d4, d4–c4.

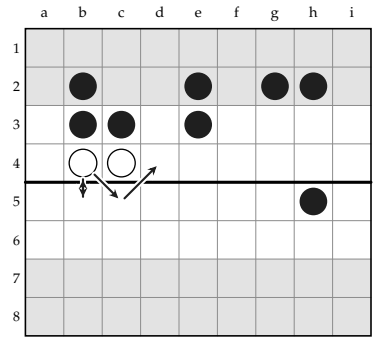
Black 12. Black takes the pearl on i1 with the two pearls nearest the corner. White now has two pearls left, so by the first rule of Section 5 Black comes out.

White 12. White moves its last two pearls to the left, away from the black pearls on the right.



	BEFORE		AFTER	
	●	○	●	○
+	0	3	0	3
x	3	0	3	0
Σ	33:9:3	8:2:0	29:8:3	8:2:0
∨	7	4	7	4

DIAGRAM 43. Black 13. c2–b3, d2–e3, f3–g4, g4–h5.

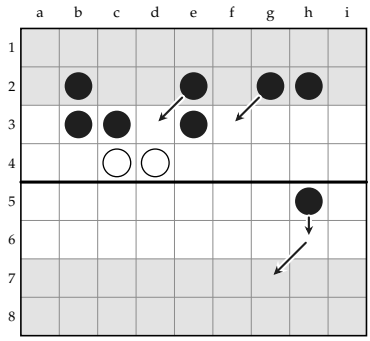


	BEFORE		AFTER	
	●	○	●	○
+	0	3	0	3
x	3	0	3	0
Σ	29:8:3	8:2:0	29:8:3	8:2:0
∨	7	4	7	4

DIAGRAM 44. White 13. b4–b5, b5–b4, b4–c5, c5–d4.

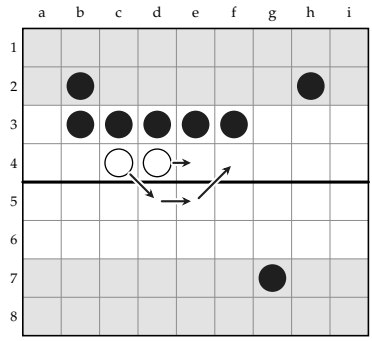
Black 13. Black comes out. One pearl starts forward on the right, and the others step back up to the third row in front of the two white pearls.

White 13. White moves its two pearls to c4 and d4. It leads and wants the game to end, but both pearls are four steps from crossing, with black pearls in front of them.



	BEFORE		AFTER	
	●	○	●	○
+	0	3	0	3
x	3	0	3	0
Σ	29:8:3	8:2:0	26:7:2	8:2:0
∨	7	4	6	4

DIAGRAM 45. Black 14. g2–f3, e2–d3, h5–h6, h6–g7.

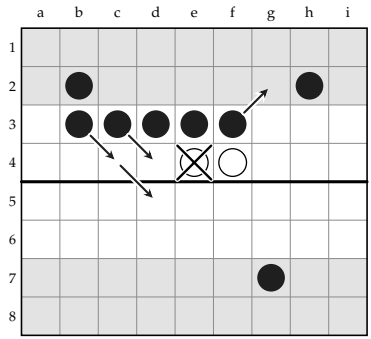


	BEFORE		AFTER	
	●	○	●	○
+	0	3	0	3
x	3	0	3	0
Σ	26:7:2	8:2:0	26:7:2	8:2:0
∨	6	4	6	4

DIAGRAM 46. White 14. c4–d5, d4–e4, d5–e5, e5–f4.

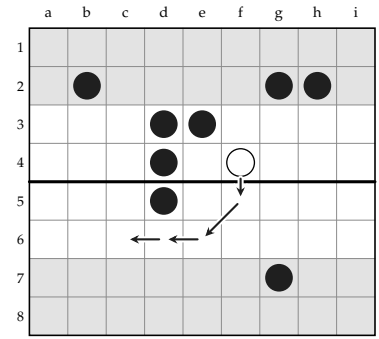
Black 14. Black's runner reaches g7. Five black pearls stand in line again on the third row.

White 14. White brings its two pearls across to e4 and f4, in front of Black's wall. This is White's last mistake.



	BEFORE		AFTER	
	●	○	●	○
+	0	3	0	3
x	3	0	4	0
Σ	26:7:2	8:2:0	17:5:3	4:1:0
∨	6	4	6	4

DIAGRAM 47. Black 15. b3–c4, f3–g2, c4–d5, c3–d4 ×e4.

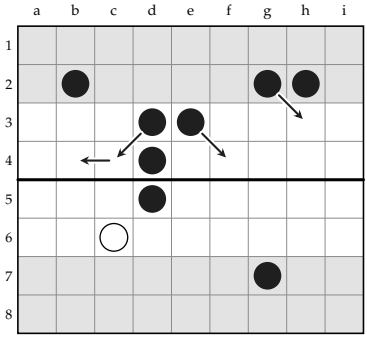


	BEFORE		AFTER	
	●	○	●	○
+	0	3	0	3
x	4	0	4	0
Σ	17:5:3	4:1:0	17:5:3	6:2:2
∨	6	4	6	6

DIAGRAM 48. White 15. f4–f5, f5–e6, e6–d6, d6–c6.

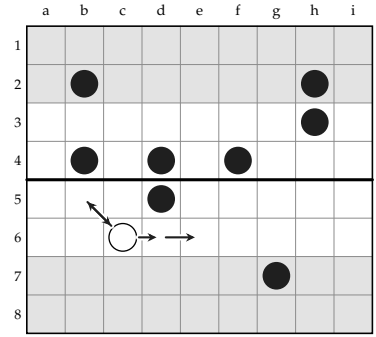
Black 15. Black takes the pearl on e4. White has one pearl left, on f4, and none of its forward squares is open to it: e3 is filled, and f3 and g3 each touch two black pearls. White can no longer cross, and so it cannot end the game by scoring.

White 15. The white pearl runs back toward its own side. Black must not take it. At three points to none, a capture would leave White with no pearls, end the game and give it to White.



	BEFORE		AFTER	
	●	○	●	○
+	0	3	0	3
x	4	0	4	0
Σ	17:5:3	6:2:2	16:4:0	6:2:2
∨	6	6	5	6

DIAGRAM 49. Black 16. g2–h3, d3–c4, c4–b4, e3–f4.

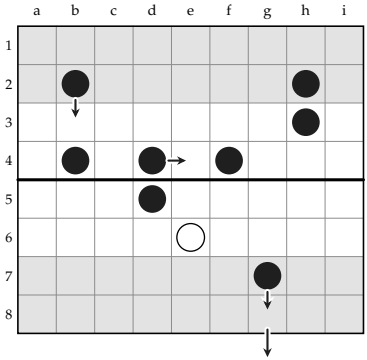


	BEFORE		AFTER	
	●	○	●	○
+	0	3	0	3
x	4	0	4	0
Σ	16:4:0	6:2:2	16:4:0	6:2:2
∨	5	6	5	6

DIAGRAM 50. White 16. c6–b5, b5–c6, c6–d6, d6–e6.

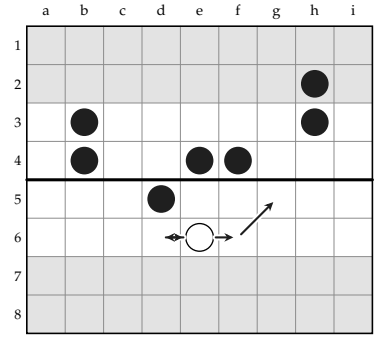
Black 16. Black goes forward on a broad front and leaves the white pearl alone.

White 16. White marks time.



	BEFORE		AFTER	
	●	○	●	○
+	0	3	1	3
x	4	0	4	0
Σ	16:4:0	6:2:2	14:4:2	6:2:2
∨	5	6	5	6

DIAGRAM 51. Black 17. b2–b3, g7–g8, g8–off, d4–e4.

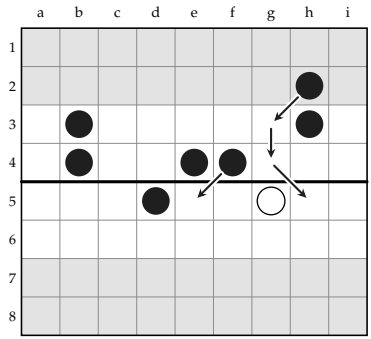


	BEFORE		AFTER	
	●	○	●	○
+	1	3	1	3
x	4	0	4	0
Σ	14:4:2	6:2:2	14:4:2	5:2:3
∨	5	6	5	5

DIAGRAM 52. White 17. e6–d6, d6–e6, e6–f6, f6–g5.

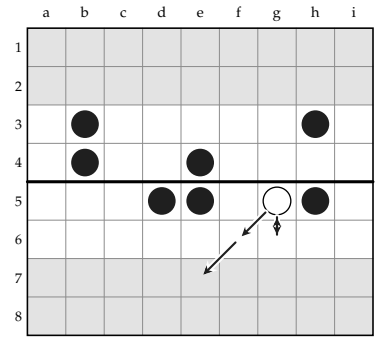
Black 17. Black crosses for the first time.

White 17. The white pearl keeps out of reach of the black pearls.



	BEFORE		AFTER	
	●	○	●	○
+	1	3	1	3
x	4	0	4	0
Σ	14:4:2	5:2:3	12:3:0	5:2:3
∨	5	5	4	5

DIAGRAM 53. Black 18. h2–g3, f4–e5, g3–g4, g4–h5.

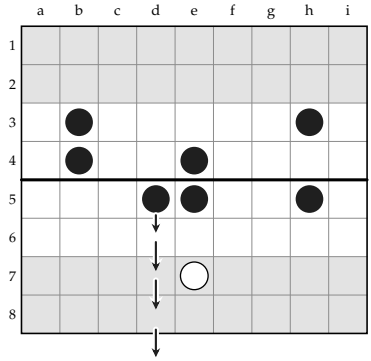


	BEFORE		AFTER	
	●	○	●	○
+	1	3	1	3
x	4	0	4	0
Σ	12:3:0	5:2:3	12:3:0	7:2:1
∨	4	5	4	7

DIAGRAM 54. White 18. g5–g6, g6–g5, g5–f6, f6–e7.

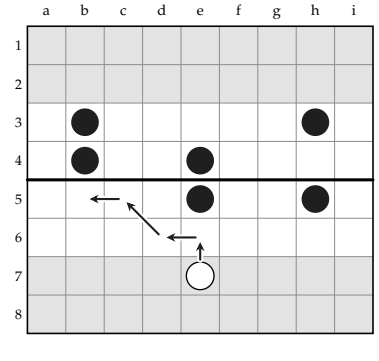
Black 18. Black brings more pearls forward.

White 18. The white pearl moves toward its own back rows.



	BEFORE		AFTER	
	●	○	●	○
+	1	3	2	3
x	4	0	4	0
Σ	12:3:0	7:2:1	8:2:0	7:2:1
∨	4	7	4	7

DIAGRAM 55. Black 19. d5–d6, d6–d7, d7–d8, d8–off.

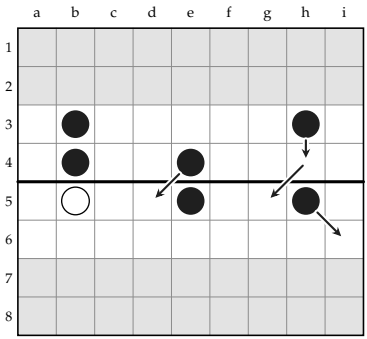


	BEFORE		AFTER	
	●	○	●	○
+	2	3	2	3
x	4	0	4	0
Σ	8:2:0	7:2:1	8:2:0	5:2:3
∨	4	7	4	5

DIAGRAM 56. White 19. e7–e6, e6–d6, d6–c5, c5–b5.

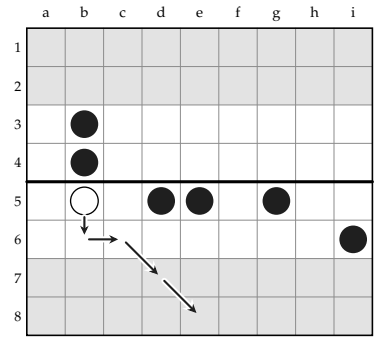
Black 19. Black crosses a second time.

White 19. The white pearl runs to the other wing.



	BEFORE		AFTER	
	●	○	●	○
+	2	3	2	3
x	4	0	4	0
Σ	8:2:0	5:2:3	7:2:1	5:2:3
∨	4	5	4	5

DIAGRAM 57. Black 20. h5–i6, h3–h4, h4–g5, e4–d5.

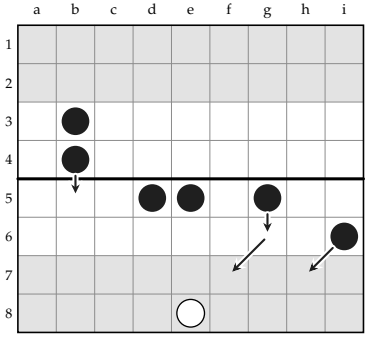


	BEFORE		AFTER	
	●	○	●	○
+	2	3	2	3
x	4	0	4	0
Σ	7:2:1	5:2:3	7:2:1	8:2:0
∨	4	5	4	8

DIAGRAM 58. White 20. b5–b6, b6–c6, c6–d7, d7–e8.

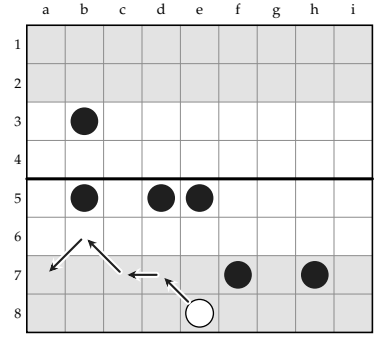
Black 20. Black moves three more pearls forward.

White 20. The white pearl goes back to its own back row, on e8.



	BEFORE		AFTER	
	●	○	●	○
+	2	3	2	3
x	4	0	4	0
Σ	7:2:1	8:2:0	4:1:0	8:2:0
∨	4	8	2	8

DIAGRAM 59. Black 21. i6–h7, g5–g6, g6–f7, b4–b5.

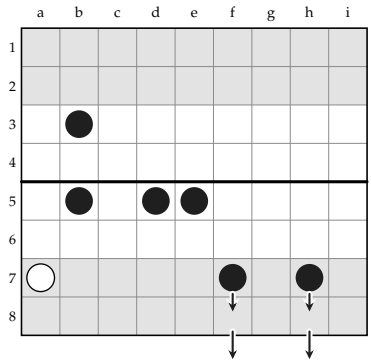


	BEFORE		AFTER	
	●	○	●	○
+	2	3	2	3
x	4	0	4	0
Σ	4:1:0	8:2:0	4:1:0	7:2:1
∨	2	8	2	7

DIAGRAM 60. White 21. e8–d7, d7–c7, c7–b6, b6–a7.

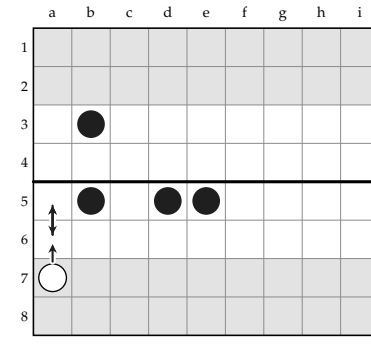
Black 21. Two black pearls stand on the seventh row, one step from crossing.

White 21. The white pearl runs to the a-file.



	BEFORE ●	BEFORE ○	AFTER ●	AFTER ○
+	2	3	4	3
x	4	0	4	0
Σ	4:1:0	7:2:1	0:0:0	7:2:1
∨	2	7	–	7

DIAGRAM 61. Black 22. h7–h8, h8–off, f7–f8, f8–off.

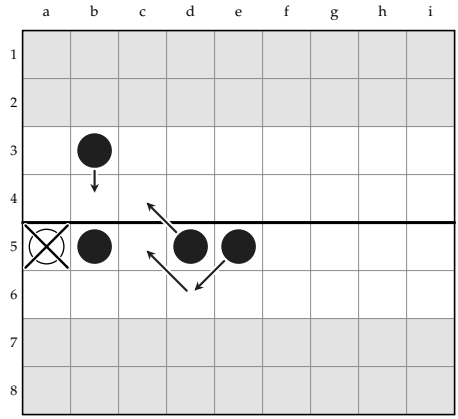


	BEFORE ●	BEFORE ○	AFTER ●	AFTER ○
+	4	3	4	3
x	4	0	4	0
Σ	0:0:0	7:2:1	0:0:0	5:2:3
∨	–	7	–	5

DIAGRAM 62. White 22. a7–a6, a6–a5, a5–a6, a6–a5.

Black 22. Black crosses twice in one turn and leads by four points to three. Now Black wins by a capture, because it would leave White with no pearls while Black is ahead.

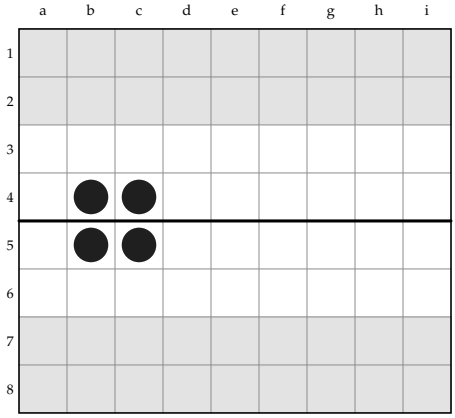
White 22. The white pearl can only move up and down the a-file.



	BEFORE ●	BEFORE ○	AFTER ●	AFTER ○
+	4	3	4	3
x	4	0	5	0
Σ	0:0:0	5:2:3	0:0:0	0:0:0
∨	–	5	–	–

DIAGRAM 63. Black 23. e5–d6, d6–c5, d5–c4, b3–b4 ×a5.

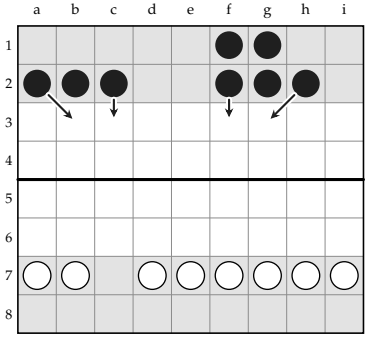
Black 23. Black surrounds the last white pearl and takes it. Black wins by four points to three.



	●	○
+	4	3
x	5	0
Σ	0:0:0	0:0:0
∨	–	–

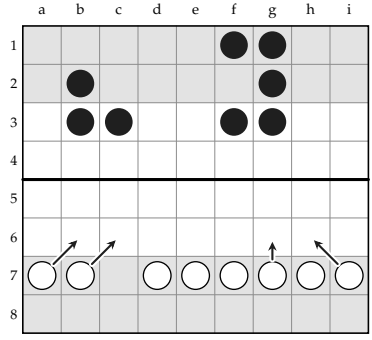
DIAGRAM 64. The final position.

Second game. White makes the same attack and loses no pearl it needs. White wins by four points to one.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	58:15:2	56:14:0	54:14:2	56:14:0
∨	8	7	8	7

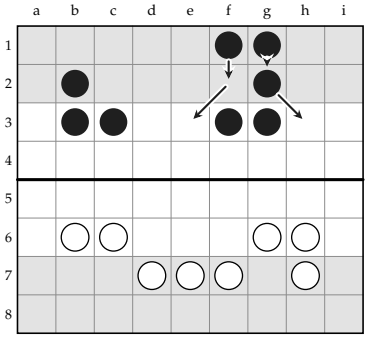
DIAGRAM 65. Black 1. h2–g3, f2–f3, a2–b3, c2–c3.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	54:14:2	56:14:0	54:14:2	52:13:0
∨	8	7	8	7

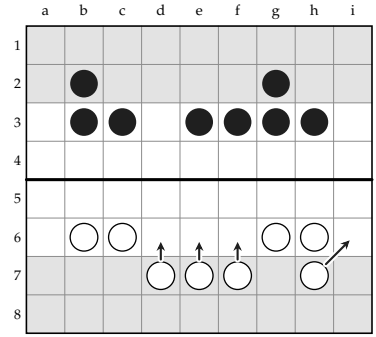
DIAGRAM 66. White 1. g7–g6, b7–c6, a7–b6, i7–h6.

Black 1. Black starts building its wall on the third row.
White 1. White comes forward on both wings.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	54:14:2	52:13:0	50:13:2	52:13:0
∨	8	7	7	7

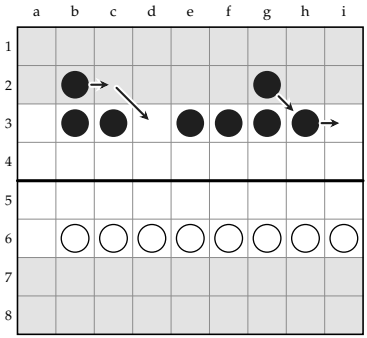
DIAGRAM 67. Black 2. g2–h3, f1–f2, f2–e3, g1–g2.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	50:13:2	52:13:0	50:13:2	48:12:0
∨	7	7	7	6

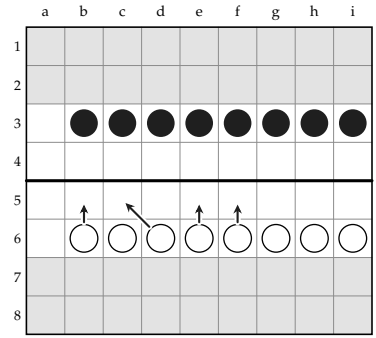
DIAGRAM 68. White 2. f7–f6, e7–e6, d7–d6, h7–i6.

Black 2. Black brings up its back pearls.
White 2. White forms a line of eight on its sixth row.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	50:13:2	48:12:0	48:12:0	48:12:0
∨	7	6	6	6

DIAGRAM 69. Black 3. b2–c2, c2–d3, h3–i3, g2–h3.

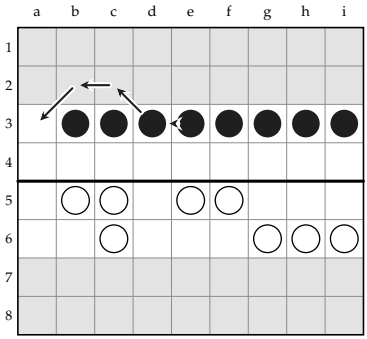


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	48:12:0	48:12:0	48:12:0	44:11:0
∨	6	6	6	6

DIAGRAM 70. White 3. f6–f5, e6–e5, d6–c5, b6–b5.

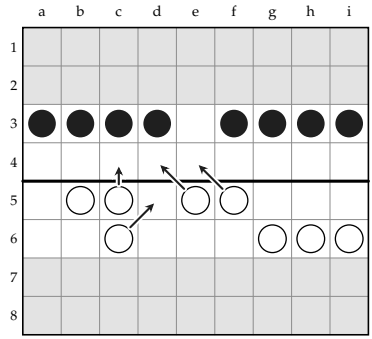
Black 3. Black completes a line of eight from b3 to i3. The a-file is open.

White 3. White brings four pearls forward to its fifth row.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	48:12:0	44:11:0	48:12:0	44:11:0
∨	6	6	6	6

DIAGRAM 71. Black 4. d3–c2, c2–b2, b2–a3, e3–d3.

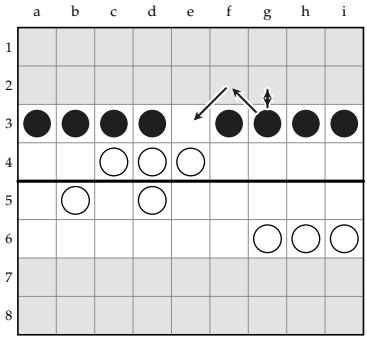


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	48:12:0	44:11:0	48:12:0	40:10:0
∨	6	6	6	6

DIAGRAM 72. White 4. c6–d5, f5–e4, e5–d4, c5–c4.

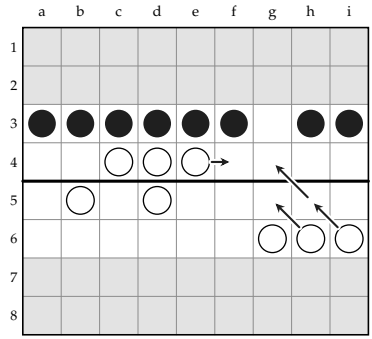
Black 4. Black moves the open file into the middle of its line, to the e-file.

White 4. White puts three pearls in front of the black pearl on d3 and threatens to take it with a fourth, as in Diagram 7.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	48:12:0	40:10:0	48:12:0	40:10:0
∨	6	6	6	6

DIAGRAM 73. Black 5. g3–g2, g2–g3, g3–f2, f2–e3.

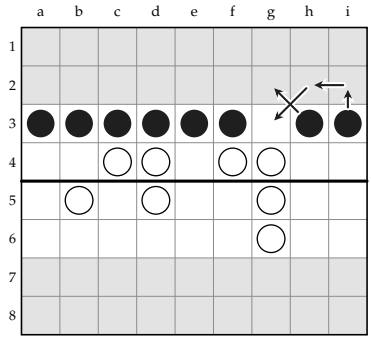


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	48:12:0	40:10:0	48:12:0	37:10:3
∨	6	6	6	6

DIAGRAM 74. White 5. i6–h5, h6–g5, e4–f4, h5–g4.

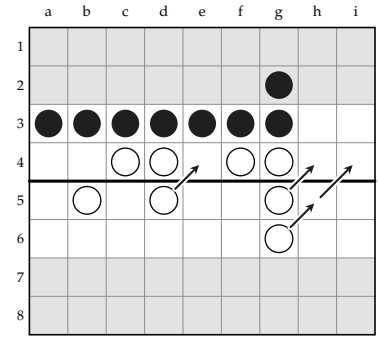
Black 5. Black moves the gap away, as in Diagram 8. The open file is now the g-file, and White has nothing in front of it.

White 5. White moves across to follow the gap.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	48:12:0	37:10:3	49:13:3	37:10:3
∨	6	6	7	6

DIAGRAM 75. Black 6. i3–i2, i2–h2, h2–g3, h3–g2.

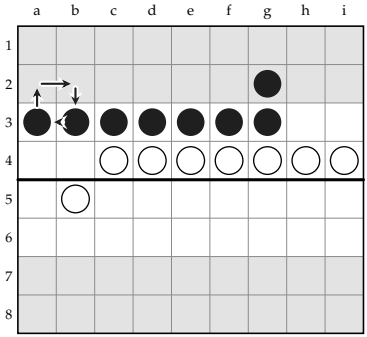


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	49:13:3	37:10:3	49:13:3	33:9:3
∨	7	6	7	5

DIAGRAM 76. White 6. g6–h5, h5–i4, d5–e4, g5–h4.

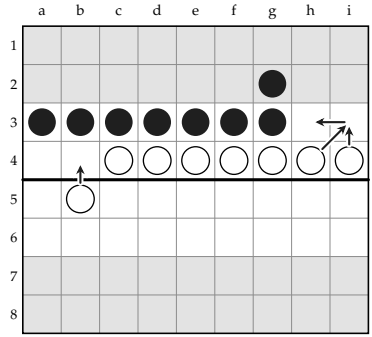
Black 6. Black forms seven and one. The pearl from i3 goes round to g3, and the pearl on h3 drops back to g2, behind the end of the line.

White 6. White completes a line of seven on its fourth row, from c4 to i4. White now has a full line in front of Black's wall, and, as Section 3 explains, Black can no longer move a gap away from it.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	49:13:3	33:9:3	49:13:3	33:9:3
∨	7	5	7	5

DIAGRAM 77. Black 7. a3–a2, a2–b2, b3–a3, b2–b3.

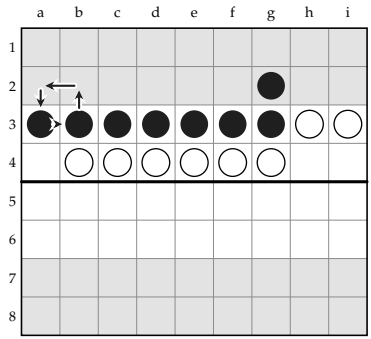


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	49:13:3	33:9:3	49:13:3	30:8:2
∨	7	5	7	4

DIAGRAM 78. White 7. i4–i3, i3–h3, b5–b4, h4–i3.

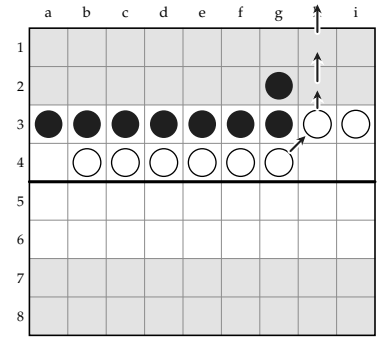
Black 7. Black marks time. This is the last turn on which Black could have acted against the attack that follows. Section 8 discusses whether a black pearl placed on h3 or i3 now would have held.

White 7. Two white pearls land together on h3 and i3. Each supports the other, so four black pearls would be needed to take either, and Black cannot bring four round them. This is the attack of Diagrams 10 and 11, in Section 4, and it decides the game.



	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	0
x	0	0	0	0
Σ	49:13:3	30:8:2	49:13:3	30:8:2
∨	7	4	7	4

DIAGRAM 79. Black 8. b3–b2, b2–a2, a3–b3, a2–a3.

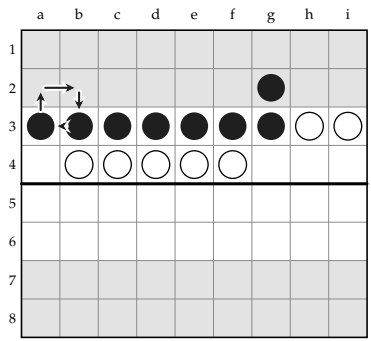


	BEFORE		AFTER	
	●	○	●	○
+	0	0	0	1
x	0	0	0	0
Σ	49:13:3	30:8:2	49:13:3	26:7:2
∨	7	4	7	4

DIAGRAM 80. White 8. h3–h2, h2–h1, h1–off, g4–h3.

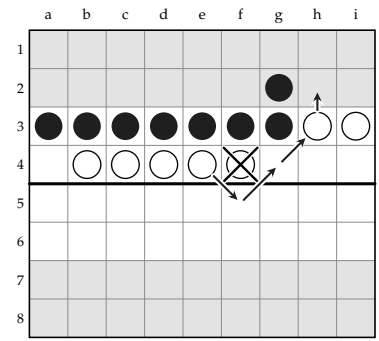
Black 8. Black marks time. As Section 4 explains, no turn Black can play here stops White from crossing without opening the wall elsewhere.

White 8. White crosses for the first time, by h2 and h1, and another pearl steps up to h3.



	BEFORE		AFTER	
	●	○	●	○
+	0	1	0	1
x	0	0	0	0
Σ	49:13:3	26:7:2	49:13:3	26:7:2
∨	7	4	7	4

DIAGRAM 81. Black 9. a3–a2, a2–b2, b3–a3, b2–b3.

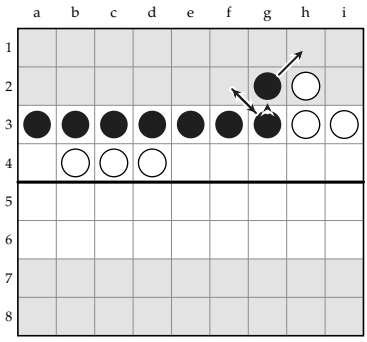


	BEFORE		AFTER	
	●	○	●	○
+	0	1	0	1
x	0	0	1	0
Σ	49:13:3	26:7:2	42:11:2	20:5:0
∨	7	4	6	4

DIAGRAM 82. White 9. e4–f5, f5–g4, h3–h2, g4–h3 ×f4.

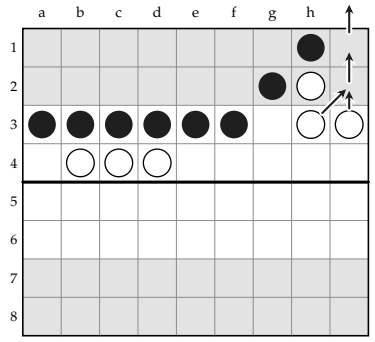
Black 9. Black marks time.

White 9. White brings a third pearl round to h3 and gives up the pearl on f4. White does not need that pearl. Giving it up lowers the target to seven and White's count from 26 to 20.



	BEFORE		AFTER	
	●	○	●	○
+	0	1	0	1
x	1	0	1	0
Σ	42:11:2	20:5:0	43:11:1	20:5:0
∨	6	4	7	4

DIAGRAM 83. Black 10. g3–f2, f2–g3, g2–h1, g3–g2.

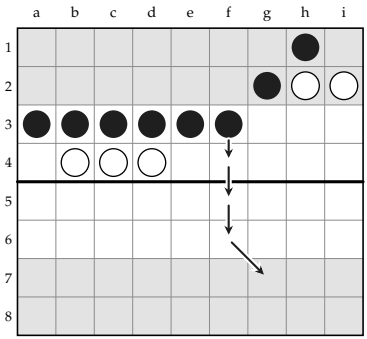


	BEFORE		AFTER	
	●	○	●	○
+	0	1	0	2
x	1	0	1	0
Σ	43:11:1	20:5:0	43:11:1	16:4:0
∨	7	4	7	4

DIAGRAM 84. White 10. h3–i2, i2–i1, i1–off, i3–i2.

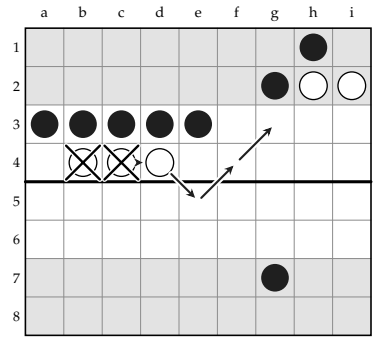
Black 10. Black puts a pearl on h1 to shut that square.

White 10. White crosses by the i-file instead and leads by two points to none. Its two pearls on the right, on h2 and i2, are each two steps from crossing.



	BEFORE		AFTER	
	●	○	●	○
+	0	2	0	2
x	1	0	1	0
Σ	43:11:1	16:4:0	39:10:1	16:4:0
∨	7	4	7	4

DIAGRAM 85. Black 11. f3–f4, f4–f5, f5–f6, f6–g7.

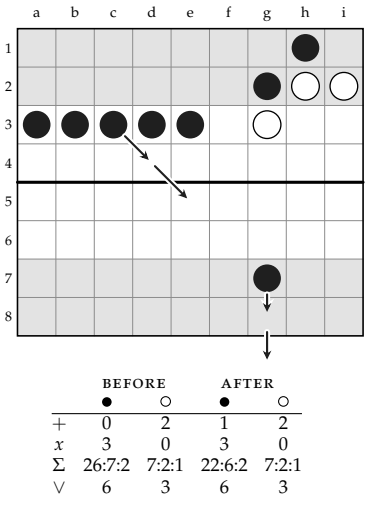


	BEFORE		AFTER	
	●	○	●	○
+	0	2	0	2
x	1	0	3	0
Σ	39:10:1	16:4:0	26:7:2	7:2:1
∨	7	4	6	3

DIAGRAM 86. White 11. d4–e5, c4–d4 ×b4, e5–f4 ×d4, f4–g3.

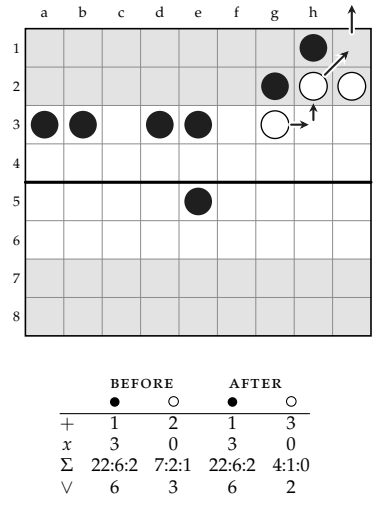
Black 11. White has scored twice, so by the second rule of Section 5 Black comes out. One pearl runs up the f-file.

White 11. White gives up two pearls on its left and brings the third across to g3. The target falls to five. White now needs two turns to finish and Black needs seven.



	BEFORE		AFTER	
	●	○	●	○
+	0	2	1	2
x	3	0	3	0
Σ	26:7:2	7:2:1	22:6:2	7:2:1
∨	6	3	6	3

DIAGRAM 87. Black 12. g7–g8, g8–off, c3–d4, d4–e5.

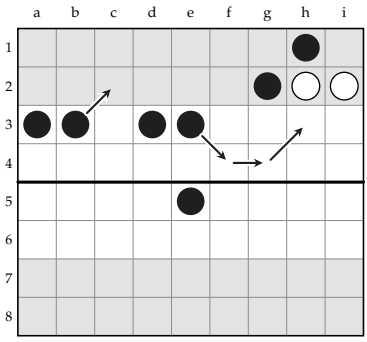


	BEFORE		AFTER	
	●	○	●	○
+	1	2	1	3
x	3	0	3	0
Σ	22:6:2	7:2:1	22:6:2	4:1:0
∨	6	3	6	2

DIAGRAM 88. White 12. g3–h3, h2–i1, h3–h2, i1–off.

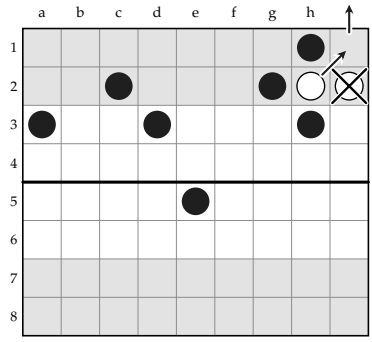
Black 12. Black crosses for the first time. It is too late, because White needs only two more turns.

White 12. White crosses a third time and leads by three points to one.



	BEFORE		AFTER	
	●	○	●	○
+	1	3	1	3
x	3	0	3	0
Σ	22:6:2	4:1:0	22:6:2	4:1:0
∨	6	2	6	2

DIAGRAM 89. Black 13. b3–c2, e3–f4, f4–g4, g4–h3.

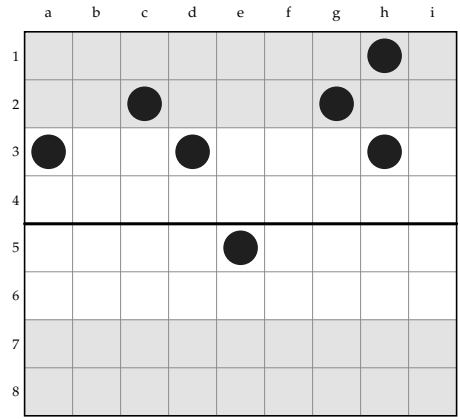


	BEFORE		AFTER	
	●	○	●	○
+	1	3	1	4
x	3	0	4	0
Σ	22:6:2	4:1:0	16:4:0	0:0:0
∨	6	2	6	–

DIAGRAM 90. White 13. h2–i1, i1–off ×i2.

Black 13. Black tries to surround White's last two pearls.

White 13. White crosses with one pearl and gives up the other. White has no pearls left, so the game ends, and White wins by four points to one. Black has seven pearls on the board and four captures, and none of them counts.



	●	○
+	1	4
x	4	0
Σ	16:4:0	0:0:0
∨	6	–

DIAGRAM 91. The final position.

Why Black won the first game. White lost five pearls in the first game, and three of them were pearls it needed. On its seventh turn it sent a pearl into the gap on e3, where Black took it once its supporting pearls had moved on. On its eleventh turn it crossed with two pearls and left a third alone in the corner on i1, where two black pearls took it. On its fourteenth turn it brought its last two pearls up in front of Black's wall, and Black took one of them. The last white pearl then had no forward square open to it. White could not cross with it, and it could not end the game by having it taken, because Black left it alone. Black needed four crossings with seven pearls and had as many turns as it wanted to make them. Black made three good decisions in the ending: it came out when White was down to two pearls, it took no pearl that would have ended the game while White was ahead, and it made the last capture only when it led by four points to three.

Why White won the second game. White never sent a pearl forward alone and never left a pearl where it could not go forward. Its attack at the end of the wall on its seventh turn was the attack of Section 4, which Black cannot stop, and from that turn Black had no way to prevent the crossings. The pearls White lost were pearls it chose to give up, on its ninth, eleventh and thirteenth turns, because it did not need them to win. Each time it gave one up, the target fell, and White needed fewer turns to finish. When White crossed for the fourth time it gave up its last pearl, and the game ended with White four points to one ahead. Black finished with seven pearls on the board and four captures, and neither counted, because the

game ended before Black could score with them. The second game shows that the waiting side's captures help it only when the opponent cannot end the game on its own terms.

Part Five
Conclusions

8. Conclusions and Open Questions

1. The waiting side keeps its pearls in a wall on its third row and marks time by exchanging two pearls. Marking time loses nothing, so the opponent cannot win by waiting for the wall to break up.
2. A gap one file wide inside the wall cannot be passed while the two pearls beside it stand. The waiting side can move the gap faster than a group of pearls can follow it.
3. Seven and one, a line of seven with a back pearl behind its end, captures every pearl that comes forward alone.
4. The end of the wall cannot be held. Two pearls that land together beside it will cross, and other pearls will follow them by the same route.
5. The waiting side should come out when the opponent has three pearls or fewer, or has scored twice.
6. The waiting side wins the ending by pinning or leaving alone the opponent's last pearl until the score allows the capture.
7. The waiting game succeeds only when the opponent loses pearls it needs in crossing, or is left with pearls that cannot go forward. Against an opponent who does neither, it fails.

Four questions remain open.

Defending the end a turn earlier. Section 4 shows that two pearls landing together beside the end of the wall cannot be stopped. It is not known whether the waiting side can stop them a turn earlier, while they are still

on the fourth row. In the second game Black marked time on the turn before they landed. A black pearl placed first on h3 or i3 would have taken one of the two squares from them. Whether that defense holds, or only moves the attack one file along, has not been worked out.

An opponent who stops attacking. If the opponent crosses once and then stops attacking, the waiting side has no good answer within the waiting game. It must come out one point behind, after many turns standing still. If both sides only mark time, the full hostile game rule, discussed in *The Level Lock*, in this volume, makes the side that began marking time change its position. A plan built on waiting makes that rule a practical matter.

A wall that is not a line. As Section 4 shows, no arrangement of eight pearls is closed to every pearl and safe from every capture. Many arrangements are closed to every lone pearl, and the line with a gap in the middle is one of them. In some of the others, the only pearl that four white pearls could surround stands on the second row, where four attackers are much harder to bring together. These arrangements are made of small groups on the first two rows. It is not known whether any of them can be reached from a real starting arrangement and held while marking time.

A deeper start. The setup rule fixes only the most pearls Black may put forward. A waiting Black may hold more pearls back, and because it does not mean to race, the steps this adds to its count matter less than usual. In B and C it also changes which side must open a level lock, unless White adjusts its own arrangement in reply, as *The Level Lock*, in this volume, discusses. Whether a deeper start makes the wall easier to build and to hold has not been worked out.

9. Further Reading

Arithmetic Margrites, in this volume, sets out the count, the edge and the spare steps that decide which side waits.

The Level Lock, in this volume, examines the position in which both sides stand in full lines at the center, and the full hostile game rule on marking time.

When to Capture, in this volume, discusses when a capture helps the side that makes it, which is the question the ending of the waiting game turns on.